Calculus 1 - Antiderivatives and Integrals

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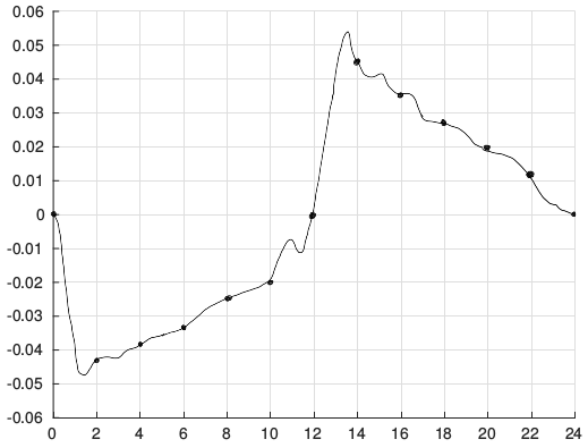
Part 1 - applications Biological activity in water is reflected in the rate at which carbon dioxide, CO_2, is added or removed. Plants take CO_2 out of the water during the day for photosyntheses and put CO_2 into the water at night. Animals put CO_2 into the water all the time as they breathe. The figure shows the rate of change of the CO_2 level in a pond where t measures the time since dawn. At dawn there was 2.600 mmol of CO_2 per liter of water.



The following table gives the rate at which CO_2 is entering or leaving the water

t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$
0	0.000	4	-0.039	8	-0.026	12	0.000	16	0.035	20	0.020
2	-0.044	6	-0.035	10	-0.020	14	0.045	18	0.027	22	0.012

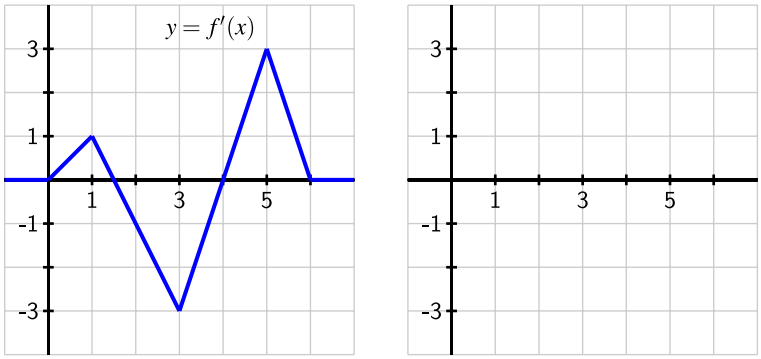
1. What are the units of $f(t)$? In words explain what would the following integral represent?

$$\int_0^3 f(t)\, dt$$

2. At what time was the CO_2 level the lowest? Highest?
3. Estimate how much CO_2 enters the pond during the night ($t = 12$ to $t = 24$).
4. Estimate the CO_2 level at dusk (twelve hours after dawn).
5. Does the CO_2 level appear to be approximately in equilibrium? (ie. after an entire day does the CO_2 remain about the same?)

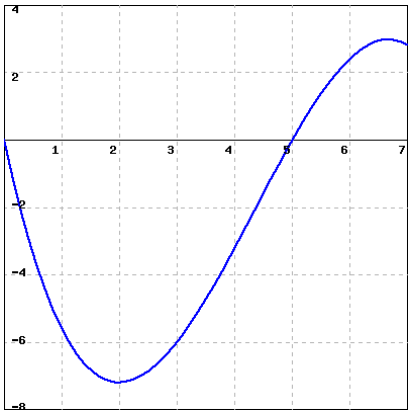
Part 2 - Drawing Antiderivatives

1. On the graph to the below you see $F(x) = f'(x)$. Using the grid to the right draw possible antiderivatives. How many antiderivatives does $f'(x)$ have?



- On what interval(s) is F an increasing function? On what intervals is f decreasing?
- On what interval(s) is F concave up? concave down?
- At what point(s) does F have a relative minimum? a relative maximum?
- Recall that
$$F(1) - F(0) = \int_0^1 f(x)\, dx$$
- If $F(0)$ =1, then what is the exact value of $F(1)$?

2. Consider the graph of the function $f(x) = F'(x)$ below and answer the following questions.



- Estimate the integral
$$\int_0^7 f(x)\, dx$$
- If the antitderivative $F(x)$ has the value $F(0) = 30$, estimate $F(7)$.
- Sketch a rough graph of the antiderivative $F(x)$ with the information given above.

Part 3 - Using Antiderivatives to Solve Integrals

For each of the integrals below, first state if the integral is a Definite or Indefinite integral then evaluate it by finding the antiderivative and using the Fundamental Theorem if it applies.

$$f(x) = x^3$$
$$g(x) = \frac{1}{x^2}$$
$$h(x) = \cos(x)$$
$$k(x) = e^x$$

1. $$\int_1^2 f(x) + 2g(x)\ dx$$

2. $$\int k(x) - 1\ dx$$

3. $$\int f(x)g(x)\ dx$$

4. $$\int_0^\pi h(x)\ dx$$

5. Why would we have trouble evaluating
$$\int_0^1 g(x)\ dx$$