Calculus 1 - Antiderivatives and Integrals – solns.

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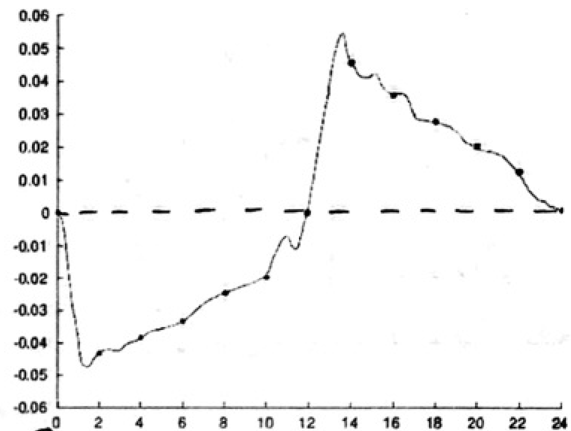
Part 1 - applications Biological activity in water is reflected in the rate at which carbon dioxide, CO_2, is added or removed. Plants take CO_2 out of the water during the day for photosyntheses and put CO_2 into the water at night. Animals put CO_2 into the water all the time as they breathe. The figure shows the rate of change of the CO_2 level in a pond where t measures the time since dawn. At dawn there was 2.600 mmol of CO_2 per liter of water.



dawn. 24 hours. $t =$ hours.

The following table gives the rate at which CO_2 is entering or leaving the water

t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$
0	0.000	4	-0.039	8	-0.026	12	0.000	16	0.035	20	0.020
2	-0.044	6	-0.035	10	-0.020	14	0.045	18	0.027	22	0.012

1. What are the units of $f(t)$? In words explain what would the following integral represent?

$$\int_0^3 f(t)\,dt$$

The integral gives the total CO_2 removed from the pond in the first 3 hours measured in mmol/liter.

$f(t)$ units mmol/liter/hr.

2. At what time was the CO_2 level the lowest? Highest?

Lowest at $t = 12$ hrs   Highest at $t = 0, 24$.

3. Estimate how much CO_2 enters the pond during the night ($t = 12$ to $t = 24$). Estimate area under curve OR use LHS, RHS.
4. Estimate the CO_2 level at dusk (twelve hours after dawn).
5. Does the CO_2 level appear to be approximately in equilibrium? (ie. after an entire day does the CO_2 remain about the same?)

#3) Try RHS. From $t=12$ to $t=24$: $\Delta t = 2$

$(.045 + .035 + .027 + .021 + .012 + 0)(2) = .28$ mmol/liter

LHS. $(0 + .45 + .035 + .027 + .021 + .012)(2) = .28$ same.

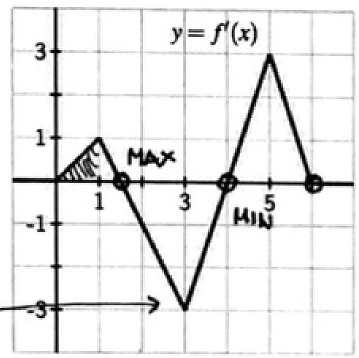
4) $F(0) = 2.6$ then subtract $(-.044 - .037 - .035 - .026 - .020 - 0)(2) = -.328$

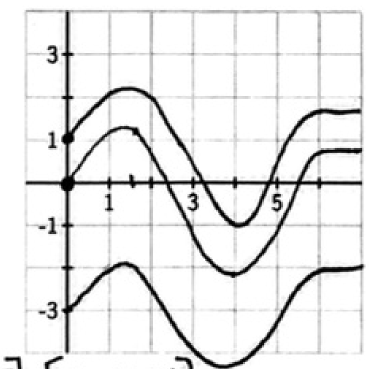
$F(12) \approx 2.272.$

Part 2 - Drawing Antiderivatives

1. On the graph to the below you see $F(x) = f'(x)$. Using the grid to the right draw possible antiderivatives. How many antiderivatives does $f'(x)$ have? There would be an infinite number of them ... ± constant.

inflections when f'' would be zero.





- On what interval(s) is F an increasing function? On what intervals is f decreasing? $[0 \leq x < 1.5]$, $[4 < x < 6]$ $[1.5 < x < 4]$
- On what interval(s) is F concave up? concave down? concave up between $[3 < x < 5]$ down $[1 < x < 3]$
- At what point(s) does F have a relative minimum? a relative maximum? MIN $x = 4$ MAX $x = 1/2$
- Recall that

$$F(1) - F(0) = \int_0^1 f(x)\,dx \qquad F(1) = F(0) + 1/2 = 1.5$$

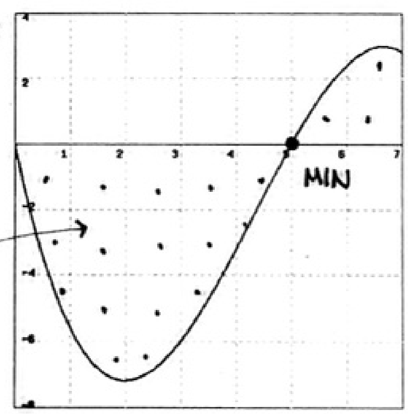
area $= 1/2$

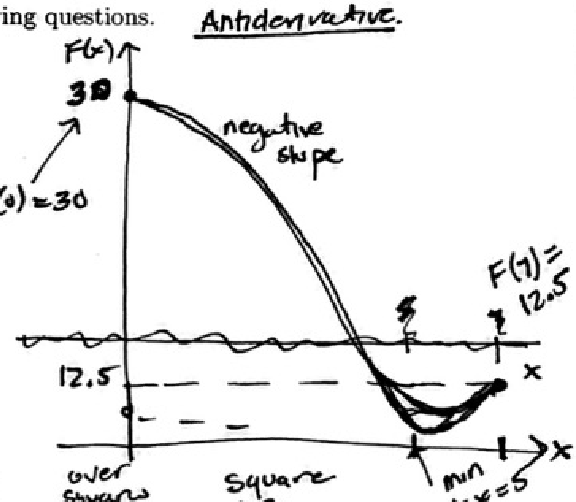
- If $F(0) = 1$, then what is the exact value of $F(1)$?

2. Consider the graph of the function $f(x) = F'(x)$ below and answer the following questions. Antiderivative.

f(x)

Area of square = 2





under squares 11.75 over squares 3 square size 4

- Estimate the integral

$$\int_0^7 f(x)\,dx = (-11.75)(2) + 3 \cdot 2 = -17.5$$

- If the antitderivative $F(x)$ has the value $F(0) = 30$, estimate $F(7)$. $F(7) - F(0) = \int_0^7 f(x)\,dx$ $F(7) = 12.5$
- Sketch a rough graph of the antiderivative $F(x)$ with the information given above.

Part 3 - Using Antiderivatives to Solve Integrals

For each of the integrals below, first state if the integral is a Definite or Indefinite integral then evaluate it by finding the antiderivative and using the Fundamental Theorem if it applies.

Antiderivatives:

$f(x) = x^3$ $\qquad F(x) = \frac{1}{4}x^4$

$g(x) = \frac{1}{x^2}$ $\qquad G(x) = -\frac{1}{x}$

$h(x) = \cos(x)$ $\qquad H(x) = \sin(x)$

$k(x) = e^x$ $\qquad K(x) = e^x$

1. Definite $\qquad \int_1^2 f(x) + 2g(x)\,dx = \int_1^2 f(x)\,dx + 2\int_1^2 g(x)\,dx = F(2) - F(1) + 2(G(2) - G(1)) = 4.75$

2. Indefinite $\qquad \int k(x) - 1\,dx = \int k(x)\,dx - \int 1\,dx = e^x - x + C$

3. Indefinite $\qquad \int f(x)g(x)\,dx = \int x\,dx = \frac{x^2}{2} + C$

do mult first!

4. Definite $\qquad \int_0^\pi h(x)\,dx = H(\pi) - H(0) = 0$

5. Why would we have trouble evaluating

$$\int_0^1 g(x)\,dx = G(1) - G(0)$$

undefined!