WORKSHEET PROBLEMS.

Math 121

Exercises on Power and Combining Rules

problem #1

Find formulas for the derivatives of the functions in problems 1-4.

1. $f(x) = \sqrt[4]{x} + \dfrac{2}{x^4}$

2. $f(x) = 8x^4 + 5^7 - 3x^{2/3}$

3. $f(x) = \dfrac{x - 1 + x^3}{x^2}$ (Hint: first use algebra to rewrite this in another form.)

4. $f(x) = (3x^2 - 1)^2$ (Hint: again, use algebra first.)

problem #2

5. **Patterns of derivatives:** In each of the following cases, take the first derivative, the second derivative, the third derivative (i.e., the derivative of the second derivative, and the fourth derivative.
   (a) $f(x) = x^3 + 2x^2 - 14x + 33$

   (b) $f(x) = x^3 - 22x^2 - 72$

   (c) $f(x) = x^3 + 387x + 90$

   (d) $f(x) = x^3 + 47x^2 - 16x$

For each problem above.

6. Look at your answers to problem 5. What do you see that they have in common? Can you explain why?

Extra →

7. **Bonus problem!** If $f(x) = x^4 + 13x^3 - 55x^2 + 8x - 129$, find the fourth and fifth derivatives of $f(x)$ **without** actually taking any derivatives. (Wow! Can this really be done?)

Problem #3

Given

$$f(x) = \frac{1}{3}x^3 - x + 1$$

Answer the following questions:

1. Where is $f(x)$ increasing?
2. Where is $f(x)$ decreasing?
3. Where is $f(x)$ concave up?
4. Where is $f(x)$ concave down?
5. Find the equation of the tangent line at $f(0) = 1$.

Problem #4

Find the derivative of the following functions:

1. $f(x) = 2e^x + x^2$
2. $f(x) = 12e^x + 11^x$
3. $f(x) = \sqrt{x} + 4^x - \frac{3}{x^3} + 14e^x$

Problem #5

4. Find the slope of the graph of

$$f(x) = 1 - e^x$$

at the point where it crosses the y-axis. Find the equation of the tangent line to the curve at that point. Find the equation of the line perpendicular to the tangent at this point (This is called the normal line).

Problem #6

5. You are walking down the street when you come across a \$100 bill. Being the responsible math student that you are, you decide to put it in savings, where it will earn interest, 10% monthly, until spring break. The value, V, of your savings account in dollars at time t, measured in months since today, is estimated by:

$$V(t) = 100(1.10)^t$$

(a) Evaluate and interpret $V(5)$.
(b) Evaluate and interpret $V'(t)$, including units.
(c) Evaluate and interpret $V'(5)$.

Problem #7

$$s(\theta) = \sqrt{4}\sin(\theta) + \theta^4$$

$$p(t) = a^t + \sin(t) - 3\cos(t)$$

Find the 1st, 2nd, 3rd, and 4th derivatives.
What patterns do you notice?

Problem #8

For each of the following functions:

- Find the equation of the tangent line at the point $x=a$
- Sketch the function and the tangent line.
- Use the tangent line to approximate $f(.2) \sim$ ___
- Is your approximation an under or over estimate?

(a) $f(x) = 3e^x + x^2$ @ $x=0$

(b) $f(x) = \sin(x) - 2x + \sqrt[3]{x}$ note $\sqrt[3]{x} = x^{1/3}$
@ $x=1$

(c) $f(x) = \cos(x) + 3^x$ at $x=0$

(d) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} - \sin(x) + \cos(x)$
at $x=1$