Math 121

Exercises on Power and Combining Rules

Find formulas for the derivatives of the functions in problems 1-4.

1. $f(x) = \sqrt[4]{x} + \frac{2}{x^4}$ $= x^{1/4} + 2x^{-4}$ so $f'(x) = \frac{1}{4}x^{-3/4} - 8x^{-5}$

2. $f(x) = 8x^4 + 5^7 - 3x^{2/3}$ so $f'(x) = 32x^3 + \emptyset - 2x^{-1/3}$

   hey sneaky! This is a constant

3. $f(x) = \frac{x - 1 + x^3}{x^2}$ (Hint: first use algebra to rewrite this in another form.)

   $f(x) = x^{-1} - x^{-2} + x$ so $f'(x) = -x^{-2} + 2x^{-3} + 1$

4. $f(x) = (3x^2 - 1)^2$ (Hint: again, use algebra first.)

   $f(x) = 9x^4 - 6x^2 + 1$ so $f'(x) = 36x^3 - 12x + \emptyset$

5. **Patterns of derivatives:** In each of the following cases, take the first derivative, the second derivative, the third derivative (i.e., the derivative of the second derivative, and the fourth derivative.

   (a) $f(x) = x^3 + 2x^2 - 14x + 33$

   $f'(x) = 3x^2 + 4x - 14$
   $f''(x) = 6x + 4$
   $f'''(x) = 6$
   $f^{(4)}(x) = 0$

   (b) $f(x) = x^3 - 22x^2 - 72$

   $f'(x) = 3x^2 - 44x$
   $f''(x) = 6x - 44$
   $f^{(3)}(x) = 6$
   $f^{(4)}(x) = 0$

   (c) $f(x) = x^3 + 387x + 90$

   etc...

   (d) $f(x) = x^3 + 47x^2 - 16x$

6. Look at your answers to problem 5. What do you see that they have in common? Can you explain why? They all have the same fourth derivative! The other derivatives are just multiples of the power!

7. **Bonus problem!** If $f(x) = x^4 + 13x^3 - 55x^2 + 8x - 129$, find the fourth and fifth derivatives of $f(x)$ **without** actually taking any derivatives. (Wow! Can this really be done?)

   $f^{(4)}(x) = 4 \cdot 3 \cdot 2 \cdot 1 \cdot x^0 + 13 \cdot 3 \cdot 2 \cdot 1 \cdot 0$ all others would be zero...
   $= 24$

   $f^{(5)}(x) = 0$

Given

$$f(x) = \frac{1}{3}x^3 - x + 1$$

Answer the following questions:

1. Where is $f(x)$ increasing?
2. Where is $f(x)$ decreasing?
3. Where is $f(x)$ concave up?
4. Where is $f(x)$ concave down?
5. Find the equation of the tangent line at $f(0) = 1$.

(1, 2) use derivative

(3, 4) use second derivative.

$f'(x) = x^2 - 1$

$x^2 - 1 = 0$ when $x = \pm 1 \rightarrow$ points where slope goes + to −

$x^2 - 1 > 0$ when $x^2 > 1$ $\quad x < -1$, $x > 1$ Increasing.

$x^2 - 1 < 0$ when $x^2 < 1$ $\quad -1 < x < 1$ Decreasing.

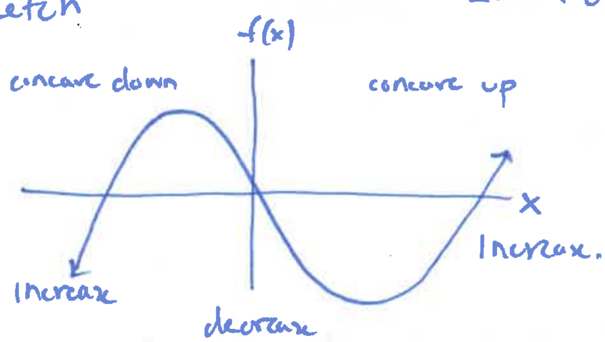
$f''(x) = 2x$

$2x = 0$ when $x = 0 \rightarrow$ point where concavity changes.

$2x > 0$ when $x > 0$ concave up

$2x < 0$ when $x < 0$ concave down.

sketch



TANGENT LINE $y = mx + b$ use $f'(x)$ to get slope at $f(0) = \phi$

$f'(0) = (0)^2 - 1 = -1$ so $m = -1$ at $(0, 1)$

$y = -x + b$ then plug in $(0, 1)$

$1 = -(0) + b$ or $b = 1$

Tangent Line: $y = -x + 1$

Graph on Desmos.

Find the derivative of the following functions:

1. $f(x) = 2e^x + x^2$

2. $f(x) = 12e^x + 11^x$

3. $f(x) = \sqrt{x} + 4^x - \frac{3}{x^3} + 14e^x$

4. Find the slope of the graph of

$$f(x) = 1 - e^x$$

at the point where it crosses the y-axis. Find the equation of the tangent line to the curve at that point. Find the equation of the line perpendicular to the tangent at this point (This is called the normal line).

5. You are walking down the street when you come across a $100 bill. Being the responsible math student that you are, you decide to put it in savings, where it will earn interest, 10% monthly, until spring break, [redacted] The value, V, of your savings account in dollars at time t, measured in months since today, is estimated by:

$$V(t) = 100(1.10)^t$$

(a) Evaluate and interpret $V(5)$.
(b) Evaluate and interpret $V'(t)$, including units.
(c) Evaluate and interpret $V'(5)$.

Handout Solutions.

1. $f(x) = 2e^x + x^2$ $\quad f'(x) = 2e^x + 2x$

2. $f(x) = 12e^x + 11^x$ $\quad f'(x) = 12e^x + \ln(11)\,11^x$

3. $f(x) = \sqrt{x} + 4^x + \frac{3}{x^3} + 14e^x$

$$f'(x) = \frac{1}{2\sqrt{x}} + \ln(4)\,4^x - \frac{9}{x^4} + 14e^x$$

4. $f'(x) = -e^x$

at the y-axis $x=0$

$$f'(0) = -e^0 = -1$$

tangent line at $x=0$ $\quad f(0) = 1 - e^0 = 1-1 = 0$

$y=0$

$y = mx + b$

$y = -x + b$ solve for b using $(0,0)$

$0 = -0 + b \quad b=0$

TANGENT

$\boxed{y = -x}$

the line perpendicular would have opposite slope!

NORMAL

$\boxed{y = x}$

5. (a) $V(5) = 100(1.10)^5 =$ how much \$ I have is 5 months.

(b) $V'(t) = 100\ln(1.10)\,1.10^t$ The rate of change of \$ per month

(c) $V'(5) = 100\ln(1.10)\,1.10^5 =$ The rate of change of \$ per month in month 5.