Exam 2 Review Problems

Economics: A certain company has the following equations for cost and revenue:

#1 $R(q) = 5q - 0.003q^2$ $C(q) = 300 + 1.1q$ q = quantity of product. where $0 \le q \le 800$

Find the quantity that maximizes the profit.

$$P = R - C = 5q - 0.003q^2 - 300 - 1.1q$$
$$= 3.9q - 0.003q^2 - 300$$

MAX or MIN:

$$P' = 3.9 - 0.006q = 0$$

so $q = \frac{3.9}{0.006} = 650$ items.

Is this a max or min?

$P'' = -0.006 < 0$ concave down $\cap$ so this is a MAX.

$q = 650$ items maximizes the profit.

Extra Problem...

Ex. For some company the fixed costs are \$4000 and the cost per item for manufacturing is given by $8q + 4q^2$. These items are sold at \$800 each. Find the quantity that maximizes the profit.

$C(q) = 4000 + 8q + 4q^2$ $R(q) = 800q$

$P(q) = 800q - 4000 - 8q - 4q^2$
$= 792q - 4000 - 4q^2$

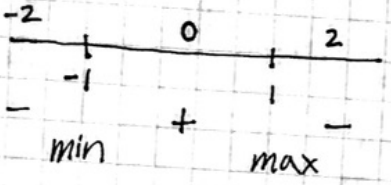
$P' = 792 - 8q$ $P'' = -8$

$q = \frac{792}{8} = 99$ this is a MAX.

Ex #2 $f(x) = \dfrac{x}{1+x^2}$ find and classify critical points global max/min and bounds.

$$f'(x) = \frac{1(1+x^2) - x(2x)}{(1+x^2)^2} = \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

where is $\dfrac{1-x^2}{(1+x^2)^2} = 0$ when $x = \pm 1$



$x = -1$ is a min
$x = 1$ is a max

then $f(-1) = \dfrac{-1}{1+1} = -\dfrac{1}{2}$ $\quad f(1) = \dfrac{1}{1+1} = \dfrac{1}{2}$

now check what happens as $x \to \pm\infty$

$$\lim_{x\to\infty} \frac{x}{1+x^2} = \lim_{x\to\infty} \frac{1/x}{1/x^2+1} = \frac{0}{0+1} = \frac{0}{1} = 0$$

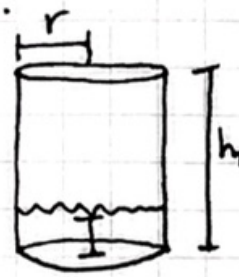
$$\lim_{x\to-\infty} \frac{x}{1+x^2} = \lim_{x\to-\infty} \frac{1/x}{1/x^2+1} = \frac{0}{0+1} = \frac{0}{1} = 0$$

So $f(-1) = -1/2$ is a global min — Upper Bound

$f(1) = 1/2$ is a global maximum. — Lower Bound

Best possible bounds: $-1/2 \le f(x) \le 1/2$

#3.



$r = 5$

$h_T = 20$ total height.

$\frac{dV}{dt} = 25\ m^3/min$ rate of water flow.

$V = \pi r^2 h = 25\pi h$ want $\frac{dh}{dt}$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$ so $\frac{dh}{dt} = \frac{1}{25\pi}\frac{dV}{dt} = \frac{1}{25\pi}(25)$

$$\boxed{\frac{dh}{dt} = \frac{1}{\pi}\ m/min.} \sim .318\ m/min$$

I am 1.5 meters tall ...

in one minute the water is at $h = \frac{1}{\pi}$
two minutes the water is at $h = \frac{2}{\pi}$

Water level = $\frac{dh}{dt} \cdot time$

$1.5 = \frac{1}{\pi} t$ $\boxed{t = 1.5\pi \text{ minutes}} \sim 4.72$ min

#4. Area = $400\ ft^2$

Supervisor Cost = $5h$ h = # of hours
Worker Cost = $10s + 5hs$ s = # of working students.

Area per student each hour = 2 so $2sh = 400$ to clean up whole yard.

$h = \frac{200}{s}$

$COST = 10s + 5hs + 5h$
$= 10s + 1000 + \frac{1000}{s}$

$\frac{dC}{ds} = 10 - \frac{1000}{s^2} = 0 \Rightarrow s^2 = 100$

$s = 10$ students

$$\text{COST} = 10(10) + 1000 + \frac{1000}{10}$$

$$= 100 + 1000 + 100 = 1200 \text{ points}$$

$$h = \frac{200}{s} = \frac{200}{10} = 20 \text{ hours.}$$

Workers $= 10 + 5(20) = 110$ points

Supervisor $= 5(20) = 100$ points.

More example problems:

Extra

Find and classify the critical points of

$$f(x) = (\sin^2 x + 2)\cos x$$

$$f'(x) = (2\sin x \cos x)\cos x + (\sin^2 x + 2)(-\sin x)$$

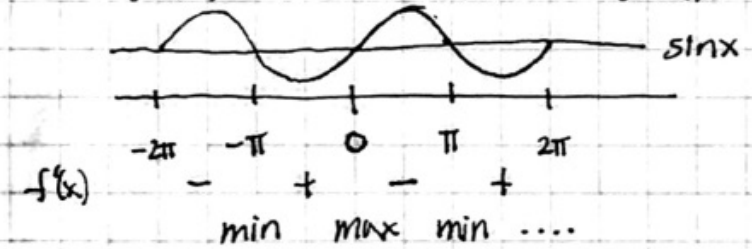
$$= 2\sin x \cos^2 x + - \sin^3 x - 2\sin x$$

but $\sin^2 x + \cos^2 x = 1$ so $\cos^2 x = 1 - \sin^2 x$

$$f'(x) = 2\sin x(1 - \sin^2 x) - \sin^3 x - 2\sin x$$

$$= -3\sin^3 x$$

$f'(x) = 0 = -3\sin^3 x$ when $x = 0 \pm \pi \pm 2\pi \pm 3\pi \ldots$.



at the critical points $\sin x = 0$ and $f(x) = (0 + 2)\cos x_c$

$= 2\cos x_c$

$= \pm 2$

Global maximum $f(x) = 2$ at $x = 0, \pm 2\pi, \pm 4\pi \ldots$.

Global minimum $f(x) = -2$ at $x = \pm\pi, \pm 3\pi, \pm 5\pi \ldots$

What are the upper and lower bounds?

Bounds $-2 \leq f(x) \leq 2$