2.3 Derivative function ...

in general we have been writing

$$f'(x) = \text{rate of change} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

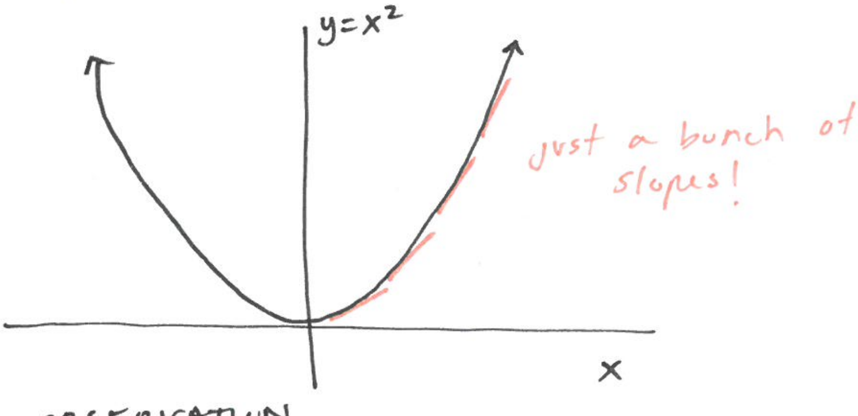
and then plugging in $x = a$ → a point.

Ex $y = x^2$ find the derivative at the following points...

$x = 1$
$x = 2$ and maybe some points inbetween.
$x = 3$
$x = 4$
⋮

hmm... this seems like a lot of work! Let's use our math brains to make this easier.

Since $y = x^2$ is a nice smooth function wouldn't the derivative EVERYWHERE be defined?



OBSERVATION

If $f(x)$ is increasing then $f'(x)$ is Positive
If $f(x)$ is decreasing then $f'(x)$ is Negative.

Lets try to find a function or formula for this derivative!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

only difference... we don't plug in a point !!

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \underbrace{h + 2x}_{} = 2x$$

Sometimes the book calls this <u>Difference Quotient</u>

so $f'(x) = 2x$.

<u>NOW</u> what is $f'(1) =$
$f'(2) =$
$f'(3) =$
$\vdots$

WAY BETTER.!!

does this make sense on the graph..
$f'(-1)$
$f'(0)$

Let's find some more of these functions.

If $f(x) = mx + b$ find $f'(x)$ .... what is your intuition on this one?

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(m(x+h)+b) - (mx+b)}{h}$$

$$= \lim_{h \to 0} \frac{mh}{h} = m$$ <u>yup</u> the slope!

so $\frac{d}{dx}[mx + b] = m$

<u>YOU TRY</u> $f(x) = k$ a constant -and- $f(x) = x^3$

$$\boxed{\text{POWER RULE} \quad f(x) = x^n \text{ then } f'(x) = nx^{n-1}}$$

Memorize This!