In two weeks...

Exam 1 Friday

~~Hand in HW8~~

~~Q's on HW8~~

~~HW 9 due Wednesday~~

DIFFERENTIATION RULES.

$$\frac{d}{dx} c f(x) = c f'(x)$$

$$\frac{d}{dx}\big(f(x) + g(x)\big) = f'(x) + g'(x)$$

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = \ln(a)\, a^x$$

$$\frac{d}{dx}\big(f(x)\cdot g(x)\big) = f'(x) g(x) + g'(x) f(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) g(x) - g'(x) f(x)}{g^2(x)}$$

Memorize These Rules!

Today: THE CHAIN RULE 3.4 allows us to take the derivative of a composite function.

$f(g(x))$ is a composite function. "Function inside a function"

Ex $(3x+2)^2$ let $f(x) = x^2$ and $g(x) = 3x+2$

then $f(g(x)) = (g(x))^2 = (3x+2)^2$

Ex e^{4x+1} let $f(x) = e^x$ and $g(x) = 4x+1$

then $f(g(x)) = e^{g(x)} = e^{4x+1}$

The Chain Rule: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x) = \frac{df}{dg} \cdot \frac{dg}{dx}$

"The derivative of the outside times the derivative of the inside"

EX $(x^3+x)^{99}$ find the derivative...

The old way... factor carefully 99 terms... uggg.

Instead use the chain rule...

$$\frac{d}{dx}(x^3+x)^{99} = \underbrace{99(x^3+x)^{98}}_{\text{derivative of outside}} \frac{d}{dx} x^3+x = 99(x^3+x)^{98} \cdot \underbrace{(3x^2+1)}_{\text{derivative of the inside.}}$$

Another way to see this $f(g(x)) = (x^3+x)^{99}$

$f(x) = x^{99}$ $f'(x) = 99x^{98}$

$g(x) = x^3+x$ $g'(x) = 3x^2+x$

now sub in: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x) = 99(x^3+x)^{98} \cdot (3x^2+x)$

What does this mean practically:

1. Your future income depends on how well you do in your classes: $I = I(G)$

   I = income (dollars) (\$) G = GPA. (tenths of a point)

2. Your GPA depends on how many hours you spend studying: $G = G(t)$ (tenths of a point) t = time (hours)

Why would we want to know $\frac{dI}{dt}$? $I = I(G(t))$

This is the change in income over the change in time studying.

If I spend one more hour studying how much will it increase my future income?

BUT $I = I(G) = I(G(t))$ composite function.

$$\frac{dI}{dt} = \frac{dI}{dG} \cdot \frac{dG}{dt}$$ chain rule. $\rightarrow$ UNITS: $\frac{\$}{\text{point}} \cdot \frac{\text{points}}{\text{hour}} = \frac{\$}{\text{hour}}$.

$\frac{dI}{dG}$: $\frac{\text{change in income}}{\text{change in GPA}}$ $\frac{dG}{dt}$: $\frac{\text{change in GPA}}{\text{change in time}}$.

At your current GPA and # of hours studying...

Say $\frac{dI}{dG} = 100$ and $\frac{dG}{dt} = .1$ what do these mean.

If my GPA goes up .01 My Income goes up by \$100

If Is spend one more hour studying my GPA will go up

Then $\frac{dI}{dt} = (100) \cdot (.1) = 100 \, \$/h$

for every hour you increase your studying your income will increase by \$100

Lots More Examples!!

<u>Ex</u> $\sqrt[3]{x^2 + 2x + 1}$

1. Simplify: $(x^2 + 2x + 1)^{1/3}$

2. Recognize outer and inner functions:
   $f(x) = \text{outer} = x^{1/3}$
   $g(x) = \text{inner} = x^2 + 2x + 1$

3. Use the chain rule

$$\frac{d}{dx}(x^2+2x+1)^{1/3} = \frac{1}{3}(x^2+2x+1)^{-2/3} \cdot (2x+2)$$

$$= \frac{(2x+2)}{3(x^2+2x+1)^{2/3}}$$

Ex $\dfrac{1}{x^2+2x+1}$ last time we would use the quotient rule on this.

1. Simplify: $(x^2+2x+1)^{-1}$ by writing this way we can use the chain rule

2. Recognize outer and inner functions
$f(x) = \text{outer} = x^{-1}$
$g(x) = \text{inner} = x^2+2x+1$

3. Use the chain rule:

$$\frac{d}{dx}(x^2+2x+1)^{-1} = -(x^2+2x+1)^{-2} \cdot (2x+2)$$
$$= \frac{-(2x+2)}{(x^2+2x+1)^2}$$

Ex $(1-e^{2t})^{19}$ sometimes we have to use the chain rule more than once.

1. Simplify ... nothing to do here

2. Recognize inner and outer
$f(t) = x^{19}$
$g(t) = 1-e^{2t}$
BUT WAIT $g(t)$ is composite $h(t) = 2t$.
really we have $f(g(h(t)))$

3. Use chain rule on all parts...

$$\frac{d}{dt}(1-e^{2t})^{19} = 19(1-e^{2t})^{18}\frac{d}{dt}(1-e^{2t})$$
$$= 19(1-e^{2t})^{18} \cdot \left(-\frac{d}{dt}e^{2t}\right)$$
$$= 19(1-e^{2t})^{18} \cdot (-e^{2t}) \cdot \frac{d}{dt}(2t)$$
$$= 19(1-e^{2t})^{18} \cdot (-e^{2t}) \cdot (2t)$$
$$= -38te^{2t}(1-e^{2t})^{18}$$

Ex $(1+xe^x)^6$ sometimes we even need to combine rules!

This is composite
but also contains a product

1. Simplify ... not needed

2. Recognize inner and outer functions along w/ product

$$f(x) = \text{outer} = x^6$$
$$g(x) = \text{inner} = 1 + xe^x$$
$$\text{product}: \; xe^x$$

3. Use rules to take derivative.

$$\frac{d}{dx}(1+xe^x)^6 = 6(1+xe^x)^5 \cdot \underbrace{\frac{d}{dx}(1+xe^x)}_{\text{look at this separately}}$$

$$\frac{d}{dx}(1+xe^x) = \frac{d}{dx}xe^x = e^x\frac{d}{dx}x + x\frac{d}{dx}e^x$$
$$= e^x + xe^x$$
$$= (1+x)e^x$$

plug back in

$$\frac{d}{dx}(1+xe^x)^6 = 6(1+xe^x)^5 \cdot (1+x)e^x$$