[redacted]

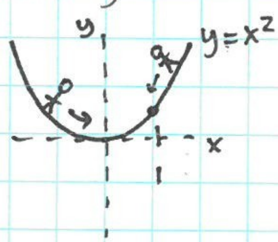
Today we work on a really important idea...

LIMITS !! notation $\lim\limits_{x \to c} f(x) = L$

What value does $f(x)$ approach as x gets really close to c?

Either it approaches ONE VALUE: L a number or the limit does not exist.

Ex a silly one $\lim\limits_{x \to 1} x^2 =$



NOTE. Taking a limit is NOT plugging in!!

walking from either side and sneaking up on $x \to 1$ we see our function $f(x) = x^2$ gets closer and closer to 1 so

$$\lim_{x \to 1} x^2 = 1$$

Ex a harder one $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} =$

what would happen if our definition was "plug in"

$\dfrac{\sin(0)}{0} = \dfrac{0}{0}$ um....

look at the graph... Desmos.

From either side we smoothly approach 1

so $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} = 1$

does this mean $\dfrac{0}{0} \Rightarrow 1$ always? NOPE... $\lim\limits_{x \to 0} \dfrac{x}{x^2}$

This is where we need to be careful and practice taking limits!

Properties to help us out...

1. If $b =$ constant $\quad \lim_{x \to c} b f(x) = b \lim_{x \to c} f(x)$

2. $\lim_{x \to c} \left(f(x) + g(x) \right) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$

3. $\lim_{x \to c} \left(f(x) \cdot g(x) \right) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$

4. $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$ as long as $\lim_{x \to c} g(x) \neq 0$

5. If $K =$ constant $\quad \lim_{x \to c} K = K$

6. $\lim_{x \to c} x = c$

Ex $\quad \lim_{x \to 3} \dfrac{x^2 + 5x}{x + 9} = \dfrac{\lim_{x \to 3} (x^2 + 5x)}{\lim_{x \to 3} (x + 9)} = \dfrac{9 + 15}{3 + 9} = \dfrac{24}{12} = 2$

note: we can approach from the left or the right and we get the same answer!

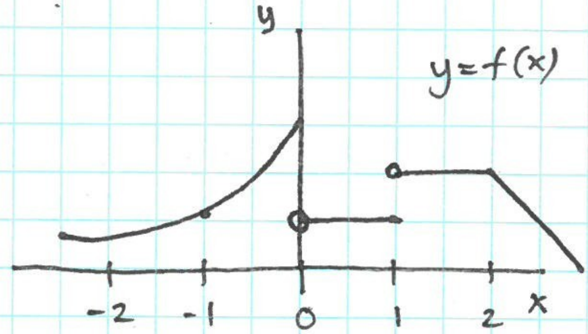
Ex $\quad \lim_{x \to \infty} \dfrac{1}{x^2}$ as x gets bigger and bigger what happens? $1/x^2 \to 0$

so the limit is 0

Ex $\quad \lim_{x \to \infty} \sin(x)$ It does not approach a single # so the limit does not exist.

We can take limits of discontinuous functions:

Ex



consider a few different limits.

$\lim_{x \to -1} f(x) = 1$ From either side!

$\lim_{x \to 0} f(x) =$ Well from the LEFT $f(x) \to 3$
RIGHT $f(x) \to 1$ DNE

this limit does not exist... BUT...

We can define the ONE SIDED LIMIT...

note limits we have been doing are TWO SIDED.

$\lim_{x \to 0^-} f(x) = 3$ From the LEFT

$\lim_{x \to 0^+} f(x) = 1$ From the RIGHT

Evaluate these limits with me:

$\lim_{x \to 1} f(x) =$ DNE $\quad \lim_{x \to 1^-} f(x) = 1 \quad \lim_{x \to 1^+} f(x) = 2$

$\lim_{x \to 2} f(x) = 2$ because it works from both sides.

Ex $\lim_{x \to 4} \frac{|x-4|}{x-4} =$ Try this one CAREFULLY

consider either side...

$\lim_{x \to 4^-} \frac{|x-4|}{x-4} \Rightarrow \lim_{x \to 4^-} \frac{-(x-4)}{x-4} = \frac{+1}{-1} = -1$ this is negative.

here $(x-4) < 0$

$\lim_{x \to 4^+} \frac{|x-4|}{x-4} \Rightarrow \lim \frac{(x-4)}{x-4} = \frac{1}{1} = 1$

here $(x-4) > 0$

So $\lim_{x \to 4} \frac{|x-4|}{x-1} = \text{DNE}$! look on graph.

Ex $\lim_{x \to \infty} \frac{x^4 + 3x}{x^4 + 2x^5}$ Trick... factor out the ∞ largest power function.

$\frac{x^4 + 3x}{x^4 + 2x^5} =$ [redacted] $\frac{x^5 (1/x + 3/x^4)}{x^5 (1/x + 2)}$ then take the limits of each piece

these cancel.

$$\frac{\lim_{x \to \infty} 1/x + \lim_{x \to \infty} 3/x^4}{\lim_{x \to \infty} 1/x + \lim_{x \to \infty} 2} = \frac{0 + 0}{0 + 2} = \frac{0}{2} = 0 \quad \checkmark$$

infinity limits always one sided.

Ex $\lim_{x \to 0} \sin(1/x)$ — look at the graph... what goes on here..

BREAK apart the function as $x \to 0$ $\frac{1}{x}$ goes to ∞

and $\lim_{x \to \infty} \sin(x) = \text{DNE}$

so we would expect

$\lim_{x \to 0} \sin(1/x) = \text{DNE}$

graph agrees!

Math Majors – work NOW to understunt the formal definitions!

Formal Definition of Limit

We define the limit as $x \to c$ of $f(x)$ to be the number L (if one exists) such that for every $\varepsilon > 0$ (as small as we want), there is a $\delta > 0$ (sufficiently small) such that if $|x - c| < \delta$ and $x \neq c$, then $|f(x) - L| < \varepsilon$.

We use the Greek letters ε (epsilon) and δ (lower case delta) in the definition of the limit.

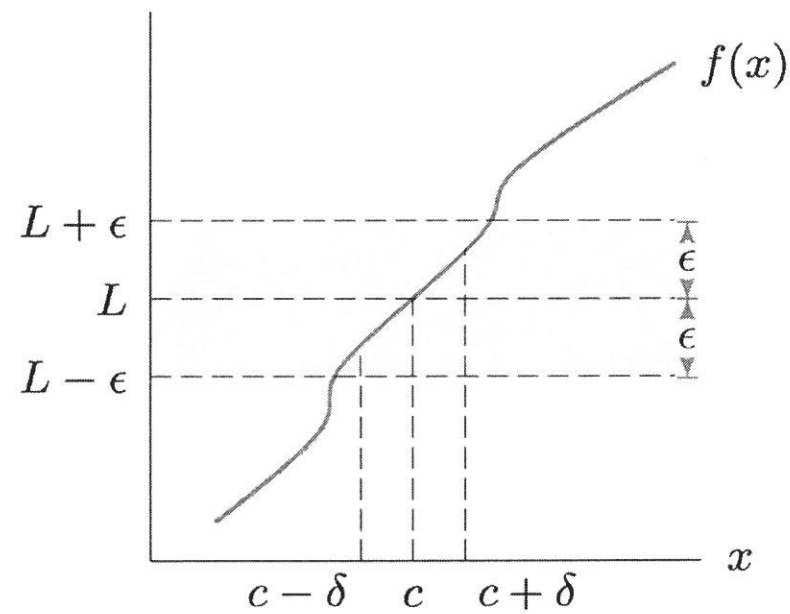


Figure 1.88: What the definition of the limit means graphically

Definition of Continuity

The function f is **continuous** at $x = c$ if f is defined at $x = c$ and if

$$\lim_{x \to c} f(x) = f(c)$$

In other words, $f(x)$ is as close as we want to $f(c)$ provided x is close enough to c. The function is **continuous on an interval** [a, b] if it is continuous at every point in the interval.

Theorem 1.3: Continuity of Sums, Products, and Quotients of Functions
Suppose that f and g are continuous on an interval and that b is a constant. Then, on that same interval,
1. $b\,f(x)$ is continuous. **2.** $f(x) + g(x)$ is continuous. **3.** $f(x)g(x)$ is continuous.
4. $f(x)/g(x)$ is continuous, provided $g(x) \neq 0$ on the interval.

Theorem 1.4: Continuity of Composite Functions
If f and g are continuous, and if the composite function $f(g(x))$ is defined on an interval, then $f(g(x))$ is continuous on that interval.