[redacted]

LAST TIME: CRITICAL POINTS: $f'(p) = 0$
the p is a critical point
$f(p)$ is a critical value.

FIRST DERIVATIVE TEST

$f'(x)$ goes from positive to negative across P then P is MAX
$f'(x)$ goes from negative to positive across P then P is MIN

SECOND DERIVATIVE TEST

$f''(P) > 0$ then f is concave up and P is a MIN
$f''(P) < 0$ the f is concave down and P is a MAX

NOTE $\longrightarrow$ $f''(p) = 0$ tells us nothing.

INFLECTION POINTS: Where $f''(x) = 0$
changes from concave up to concave down. (or vice-versa)

Ex $f(x) = x + \frac{1}{x}$ $x \neq 0$

Test for local max, min then graph the function.

$$f'(x) = 1 - \frac{1}{x^2}$$

1. Take derivatives
$f'(x)$
$f''(x)$

$$f''(x) = \frac{1}{x^3}$$

2. Critical points: $f'(x) = 1 - \frac{1}{x^2} = 0$

$\frac{1}{x^2} = 1 \qquad x^2 = 1 \qquad x = \pm\sqrt{1} = \pm 1$

critical values $f(1) = 2 \qquad f(-1) = -2$

3. Test for max or min. (Either test will work.)

FIRST DERIVATIVE

	$x = 1/2$	$x = 1$	$x = 2$
$f' =$	-3	0	$3/4$

MIN

	$x = -2$	$x = -1$	$x = -1/2$
$f' =$	$\frac{3}{4}$	0	-3

MAX

SECOND DERIVATIVE

Plug into $f''(x) = \frac{1}{x^3}$

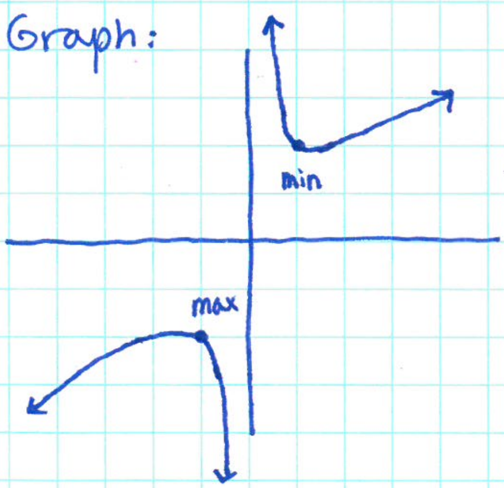
$f''(1) = \frac{1}{1^3} = 1 > 0$ MIN

$f''(-1) = \frac{1}{-1^3} = -1 < 0$ MAX

4 Inflection points $f'' = \frac{1}{x^3} = 0$ NO!

no inflection points.

5. Graph:



what happens as $x \to 0$ and $x \to \pm\infty$

TAKE LIMITS!

$$\lim_{x \to 0^+} x + \frac{1}{x} = \lim_{x \to 0^+} x + \lim_{x \to 0^+} \frac{1}{x}$$
$$= \infty$$

$$\lim_{x \to 0^{-1}} x + \frac{1}{x} = \lim_{x \to 0^-} x + \lim_{x \to 0^-} \frac{1}{x}$$
$$= -\infty$$

$$\lim_{x \to \infty} x + \frac{1}{x} = \infty$$

$$\lim_{x \to -\infty} x + \frac{1}{x} = -\infty$$

We can sketch our functions without a calculator!

4.2 Optimization.

* Local Maximum or Minimum – means the point is greater or less than the points near by.

* Global Maximum or Minimum – means the point is the largest or smallest of all points.

NOTE. If $f(x)$ is a continuous function on a closed interval $a \le x \le b$ then the global maximum and minimum must exist.

EX

Global Max and Min
Occurs either @ critical points or at end points.

This indicates a method for finding Global Max and Min:

1. Find all Critical Points and Compute Critical Values
2. Find function values at the ends of the interval
3. Compare these to find the greatest and smallest
   Sometimes this means taking a limit.

Ex Find the Global Maximum and Minimum of

$$f(x) = x^3 - 9x^2 - 48x + 52 \quad \text{on the interval } -5 \le x \le 12$$

1. Critical POINTS and VALUES

$$f'(x) = 3x^2 - 18x - 48 = 3(x^2 - 6x - 16)$$

$$= 3(x-8)(x+2) = 0 \qquad x = 8 \qquad x = -2$$

Then $f(8) = -396$ $\quad f(-2) = 104$

2. END POINTS $x = -5$ and $x = 12$

$$f(-5) = -58 \quad \text{and} \quad f(12) = -92$$

3. COMPARE VALUES

$f(-5) = -58$
$f(-2) = 104$ ✓ MAXIMUM
$f(8) = -396$ ✓ MINIMUM
$f(12) = -92$

why do I not need to do more tests?

<u>EX</u> SAME FUNCTION $f(x) = x^3 - 9x^2 - 48x + 52$
new interval: $-5 \le x < \infty$

STILL HAVE
$f(-5) = -58$
$f(-2) = 104$
$f(8) = -396$

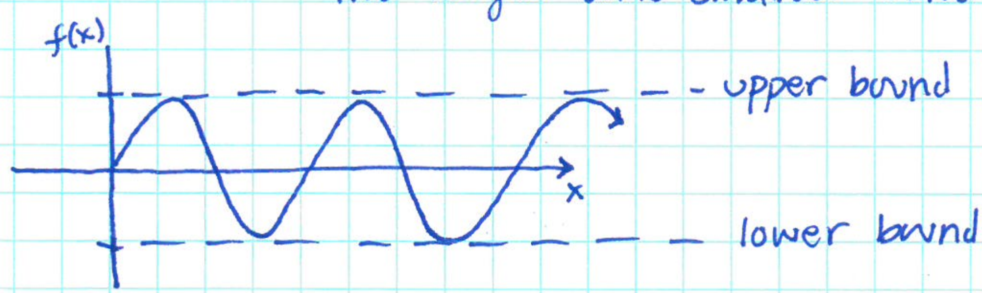
How do we find the value @ ∞? TAKE THE LIMIT

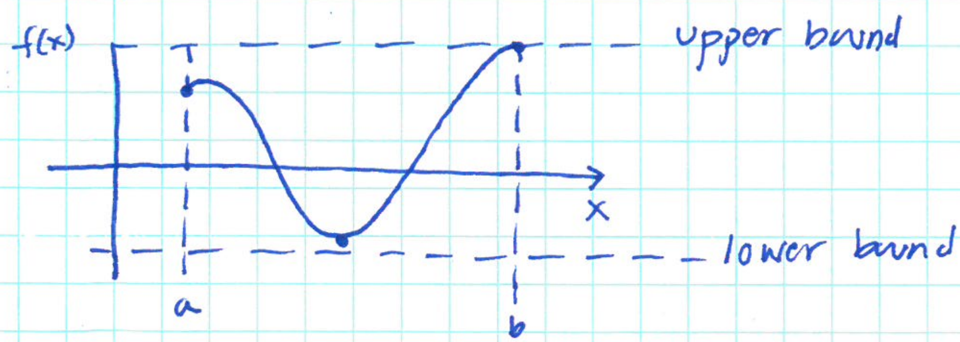
$$\lim_{x \to \infty} x^3 - 9x^2 - 48x + 52 = \infty$$

FOR THE NEW INTERVAL
there is NO GLOBAL MAX.

IMPORTANT IDEA → <u>UPPER</u> AND <u>LOWER</u> BOUNDS.

The largest and smallest values for $f(x)$





THESE COME FROM THE FUNCTION VALUES AT THE <u>GLOBAL</u> MAX and MIN.