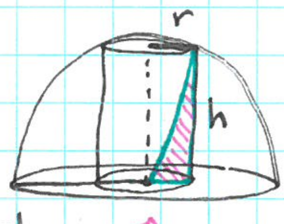
Ex Hemisphere $R=1$ sits on horizontal plane. A cylinder stands inside with its vertical, the center of it's base, at the center of the sphere, and its top circular rim touching the sphere. Find the radius that maximizes the volume. What is the height? of the cylinder

1. DRAW:



Hemisphere ✓

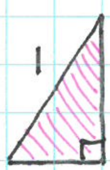
Cylinder – gets its height from the hemisphere.
"Restricted by hemisphere"

2. What we know ... RESTRICTION.

well $R=1$ how does this translate to the Cylinder?

height and radius restricted ... triangle:

in blue on shape above...



so I know $r^2 + h^2 = 1^2$

or $\boxed{r^2 = 1 - h^2}$

3. What do we want ... MAX VOLUME

Volume of Cylinder $= \pi r^2 h = V$ $\qquad \boxed{V = \pi r^2 h}$

4. Put equations together:

$$V = \pi \underbrace{(1-h^2)}_{r^2 = 1-h^2} h = \pi (h - h^3)$$

$$V' = \frac{dV}{dh} = 0$$

5. Find MAX using Calculus: $V' = \pi(1 - 3h^2) = 0$ critical points

$1 - 3h^2 = 0$

$h = \pm \frac{1}{\sqrt{3}}$

TEST: $h=0$ $\qquad$ CP $\frac{1}{\sqrt{3}}$ $\qquad$ $h=1$

$V'\big|_{h=0} = \pi$ $\qquad$ $V'\big|_{h=1} = -2\pi$

/ MAX. \

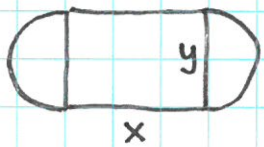
GLOBAL $h \to 0$ $V \to 0$

$h \to 1$ $r \to 0$ $V \to 0$ YES

6 Answer the Question ... $h = \frac{1}{\sqrt{3}}$ so $r = \sqrt{1-h^2} = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$ $\qquad r = \sqrt{\frac{2}{3}}$

EX Area made up of Rectangle w/ half circles on two opposite sides. Find formulas for AREA and PERIMETER. Maximize area if perimeter is 100 m.

1. DRAW



→ circles must have $r = y/2$

$$\text{AREA} = \underbrace{\frac{\pi y^2}{4}}_{\substack{\text{circle} \\ \pi r^2}} + \underbrace{xy}_{\text{rectangle}}$$

$$\text{PERIMITER} = \underbrace{2\pi\left(\frac{y}{2}\right)}_{\substack{\text{circle} \\ 2\pi r}} + \underbrace{x + x}_{\text{rectangular sides}} = \pi y + 2x$$

MAXIMIZE AREA

2. Restriction: $\boxed{\pi y + 2x = 100}$

3. What we want: MAX Area $\boxed{A = \frac{\pi y^2}{4} + xy}$

4. Put these together: $x = 50 - \frac{\pi y}{2}$ sub into A ...

$$A = \frac{\pi y^2}{4} + \underbrace{\left(50 - \frac{\pi y}{2}\right)}_{x} y = 50y - \frac{\pi y^2}{4}$$

5 MAX using calculus:

$A' = \frac{dA}{dy} = 0$ $\qquad A' = 50 - \frac{\pi y}{2} = 0 \qquad y = \frac{100}{\pi}$

TEST: $A'' = -\frac{\pi}{2} < 0$ so

concave down

thus MAX

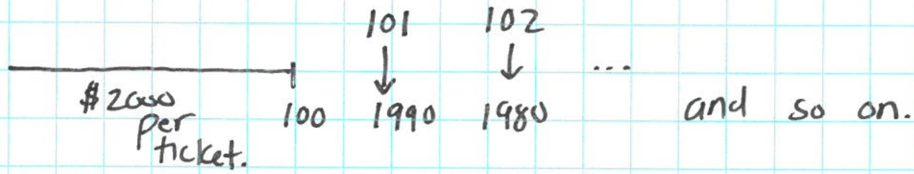
6. Answer question: $y = \frac{100}{\pi}$ maximizes the area so the maximum area must be

$$A = 50\left(\frac{100}{\pi}\right) - \frac{\pi}{4}\left(\frac{100}{\pi}\right)^2 = \frac{2500}{\pi} \checkmark$$

NOTE $x = 0$ the max area reduces problem to a circle!

EX A cruise line charges \$2000 per passenger for a ticket, but if more than 100 people sign up they discount ALL tickets by \$10 per additional customer. What is the maximum possible revenue?

1 DRAW...



2. Given information... we know the pricing structure

Ticket price =	2000	$n=100$	$n=\#$ of passengers.
	1990	$n=101$	
	1980	$n=102$	

Why don't I care about $n<100$? I know this won't maximize because adding another always increases!

$$\text{Ticket price} = \left(2000-10(n-100)\right)$$

3. What do we want? MAX REVENUE.

$$\text{Revenue} = \left(\text{Ticket Price}\right)\left(\#\text{ of passengers}\right) = \Big(\underbrace{2000-10(n-100)}_{\text{Ticket price.}}\Big)n$$

$$R = 2000n - 10n^2 + 1000n$$

$$\boxed{R = 3000n - 10n^2}$$

4. Put eqns together

5. Find MAX using calculus: $R' = \left(\frac{dR}{dn}\right) = 0$

$$R' = 3000 - 20n = 0$$

$$n = \frac{3000}{20} = 150 \text{ passengers}$$

TEST: $R'' = -20 < 0$ $\bigcap$ MAX

6. Answer Question:

Max Revenue when $n=150$

$$R = 3000(150) - 10(150)^2 = \$225{,}000$$

more or fewer passengers and they will make less money!