4.6 Rates and related rates:

* Derivatives = slope, Rate of Change ... use this when given "rate" information in a story problem!

Ex. A spherical snowball is melting. Its radius decreases at a constant rate of 2cm per minute from an initial value of 70 cm. How fast is the VOLUME decreasing 30min after melting begins.

1. Draw a picture:

r = radius

2. What we know: radius is decreasing... eqn for this.

$$r = -2t + 70$$

$70 \rightarrow$ initial radius = 70

$-2 \uparrow$ rate of <u>decrease</u> = 2

3. What do we want to know? How FAST is the volume decreasing?

$$\text{VOLUME} = V = \frac{4}{3}\pi r^3$$

4. put V and r together!

$$V = \frac{4}{3}\pi (70 - 2t)^3$$

5. Use calculus to answer the question.

How <u>FAST</u> $\Rightarrow$ take derivative

$$V' = \frac{4}{3}\pi \, 3(70 - 2t)^2 \cdot (-2) = -8\pi (70 - 2t)^2$$

6. Answer Question $\nearrow$ at $t = 30$

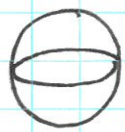
$$V'\Big|_{t=30} = -8\pi(70 - 2(30))^2 = -800\pi \ \text{cm}^3/\text{min.}$$

<u>UNITS</u>

- Sometimes we can't find an equation for what we <u>KNOW</u> but we can find an equation for the DERIVATIVE.

Related Rates... eg. The change in radius is related to the change in volume.

<u>Ex</u> A spherical snowball melts in such a way that at the instant its radius is 20 cm $(r=20)$ the radius is decreasing at a rate of 3 cm/min. At what rate is the Volume decreasing in that instant?

1. DRAW  r=radius

2. What do we know — given info:

when $r=20$ $\frac{dr}{dt} = -3$ cm. r is a function of time!

3. What do we want? — rate of change of volume!

$$V = \frac{4}{3}\pi r^3 \quad \text{and} \quad r = r(t)$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$ implicit derivative!

5. Use calculations we already did!

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

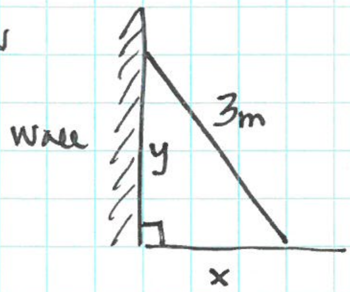
4. Plug in what we know

$$\left.\frac{dV}{dt}\right|_{r=20} = 4\pi(20)^2\left[-3\right] = -4800\pi \text{ cm}^3/\text{min.}$$

6. Answer the question... $\left.\frac{dV}{dt}\right|_{r=20} = -4800\pi \text{ cm}^3/\text{min.}$

EX A 3m long ladder leans against a wall. The foot of the ladder starts slipping outward, moving at a speed of 0.1 m/s when the foot is 1m from the wall. In that instant, how fast is the top of the ladder falling?

1 DRAW



How might we relate x and y?

$$x^2 + y^2 = 9 \quad ✓$$

2. What do we know.

Foot of ladder is at x

$\frac{dx}{dt} = 0.1$ m/s → speed of movement for the foot of ladder!

when $x = 1$ m

3. What do we want?

Top of ladder is at y.

WANT $\frac{dy}{dt}$ — as it relates to $\frac{dx}{dt}$

$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(9) \qquad 2x\frac{dx}{dt} + 2y\underline{\underline{\frac{dy}{dt}}} = 0$$

want this...

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

5. USE CALCULUS.

4. Plug in: $x = 1$ m $\quad \frac{dx}{dt} = 0.1$ m/s what is y?

$$y = \sqrt{9-x^2} = \sqrt{9-1} = \sqrt{8}$$

$$\frac{dy}{dt} = \frac{-1}{\sqrt{8}}(0.1) = -.089 \text{ m/s}$$

6. Answer: Top is moving at a speed of $-.089$ m/s ✓