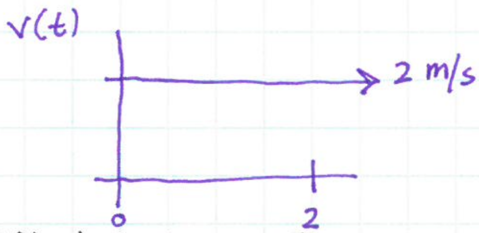
Start Ch. 5. The Definite Integral

- Derivatives ... Motivated by the idea of calculating velocity

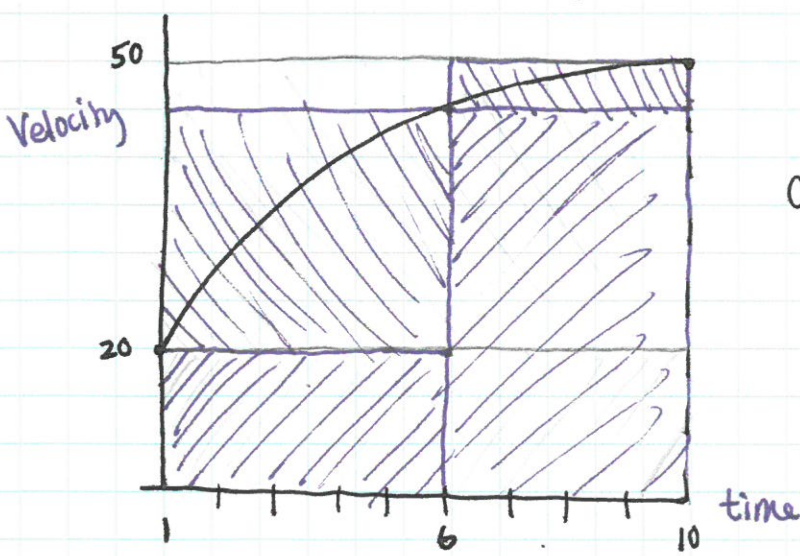
We started with an approximate or average velocity and by narrowing our change in time got to instantaneous velocity and the derivative.

- Integration ... We start by saying we know the velocity and we want to calculate the distance traveled...



The reverse problem — this will lead us to integration.

Worksheet: Do p1 together...



distance = speed x time.

Our distance calculation is the area of the rectangles...

Second try... our distance is the sum of the two areas...

Calculus 1 - Distance Traveled

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The following table gives information about the speed (velocity) of a car, measured in feet per second, as a function of time. Our goal is to estimate the distance that the car travels in the 10 seconds for which we have data.

Time	0	1	2	3	4	5	6	7	8	9	10
Velocity	20	26	30	34	38	41	44	46	48	49	50

Remember: Distance = speed × time

PART 1

1. What if we only knew the speed at $t = 0$? How far would we say the car travels in 10 seconds?

   $V=20$ when $t=0$

   so $D = 20 \times 10 = 200$ feet

2. What if we only knew the speed at $t = 10$? How far would we say that car had traveled in 10 seconds?

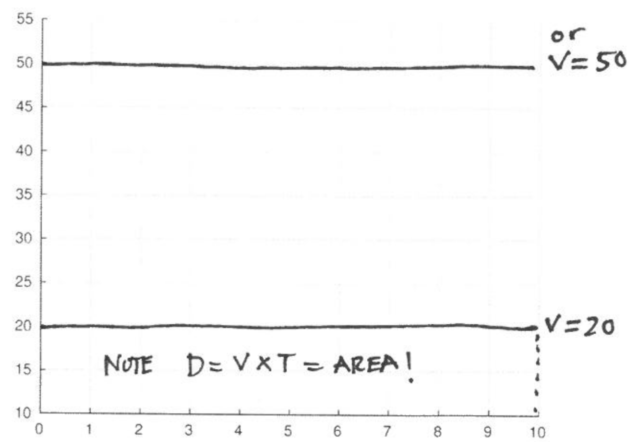
   $V=50$ when $t=10$

   so $D = 50 \times 10 = 500$ feet

3. Why are neither of the first two calculations very accurate in predicting the real distance that the car traveled?

   We assume a constant speed... and really the speed is increasing.

Draw a graph of the velocity over time that **you used** in this calculation.



$500 - 200 = 300$ ft

PART 2

1. What if now we knew the velocity at $t = 0, 5, 10$? How would we get a better estimate for the actual distance traveled? Estimate the distance:

compute two time periods:

$D = V_1 \times 5 + V_2 \times 5$

Choices...

1. $D = 20 \times 5 + 41 \times 5 = 305$
2. $D = 41 \times 5 + 50 \times 5 = 455$

2. How did you choose what speed to assume for each time period?

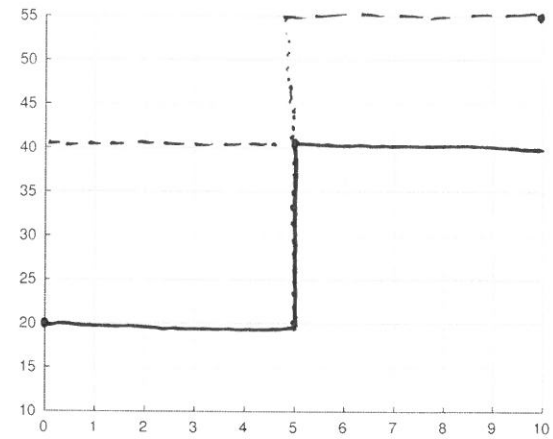
1. Assumed the slower speed or that I use $t=0$ $t=5$ ———

2. Assumed the faster speed or that I use $t=5$ $t=10$ - - - - -

3. Is your estimate and overestimate or underestimate?

1. Underestimate
2. Overestimate.

Draw a graph of the velocity over time that **you used** in this calculation.



$405 - 355 = 150$ feet

PART 3

Now using all of the give data:

1. Give an **underestimate** for the distance traveled.
2. Give and **overestimate** for the distance traveled.

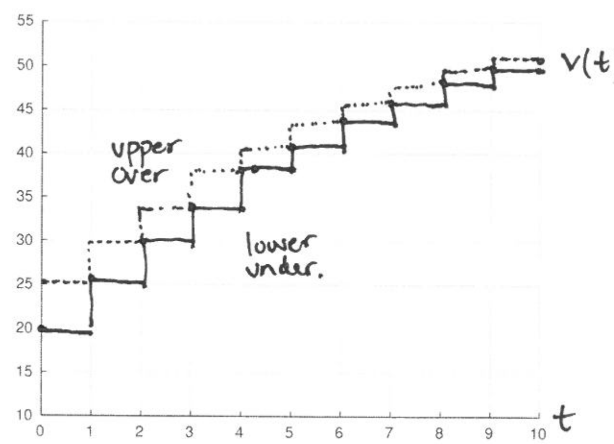
under:

$D = 20(1) + 26(1) + 30(1) + 34(1) + 38(1) + 41(1) + 44(1) + 46(1) + 48(1) + 49(1) = 376$ feet.

over

$D = 26(1) + 30(1) + 34(1) + 38(1) + 41(1) + 44(1) + 46(1) + 48(1) + 49(1) + 50(1) = 406$ feet.

Draw a graph of the velocity over time that **you used** for these calculations.



What is the difference between your underestimate and your overestimate?

$406 - 376 = 30$ feet.

PART 4

Finally, can you propose how we might go about getting an even more accurate measure for the distance traveled?

We could either get more speed data or assume linear across the top of boxes...

What additional data would we need?

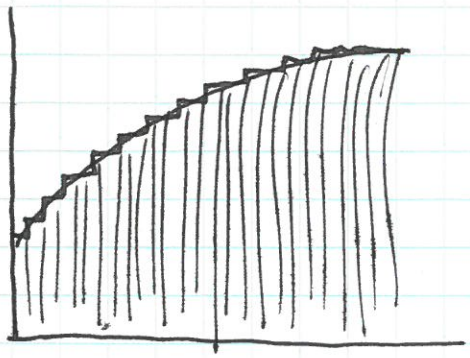
Ideally we get more speed data

How could we get an <u>exact</u> value for the distance traveled?

We would need "infinite" or continuous speed data.

After the worksheet → w/ 20 min left...

We can imagine making our time steps as narrow as possible...

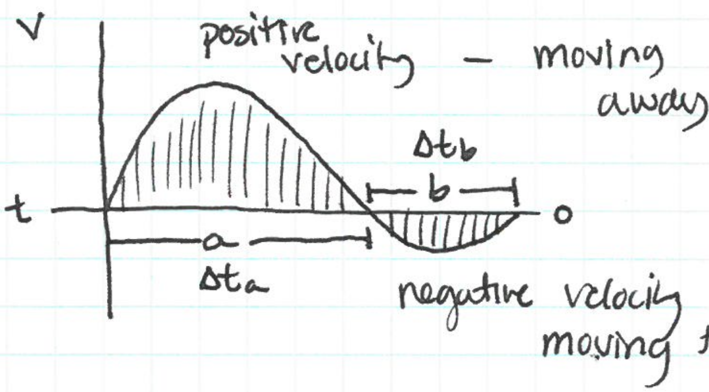


This would give the BEST estimate

So we say

If the VELOCITY is positive the distance traveled is the area under the velocity curve.

What about a velocity curve like this?



distance = speed × time

• V in section a is positive • V in section b is negative.

distance = $+V\Delta t_a + -V\Delta t_b = V\Delta t_a - V\Delta t_B$

we are adding the area under the curve the fact that V is negative takes care of the fact that we loose distance during Δt_b.

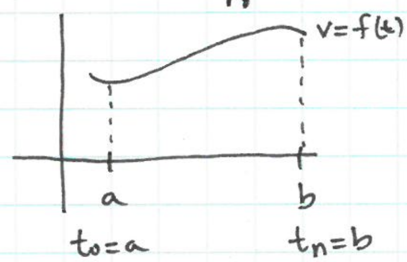
Left and Right Sums:

Writing the distance traveled between $t=a$, $t=b$ in a new notation.

call Δt the time interval between measurements. $\rightarrow$ $\Delta t = \dfrac{b-a}{n}$

$n = \#$ of subdivisions. between a and b

say $f(t)$ = the velocity at time t.



Then for the lower estimate:

the distance traveled in

the 1^{st} time period: $f(t_0)\Delta t$

the 2^{nd} time period: $f(t_1)\Delta t$

RIGHT HAND SUM = ~~lower estimate of the total distance traveled~~ $\cong f(t_0)\Delta t + f(t_1)\Delta t + f(t_2)\Delta t + \cdots + f(t_{n-1})\Delta t$

- we separated the graph into n time values.

LEFT HAND SUM = ~~the upper estimate of the total distance traveled~~ $\cong f(t_1)\Delta t + f(t_2)\Delta t + \cdots + f(t_n)\Delta t$

Accuracy of Estimates = | Difference Between Upper and Lower estimates | = | Difference between $f(a)$ and $f(b)$ | $\cdot \Delta t$

$= |f(b) - f(a)| \cdot \Delta t.$

UNDER or Overestimate depends on $f(t)$ increasing or decreasing.