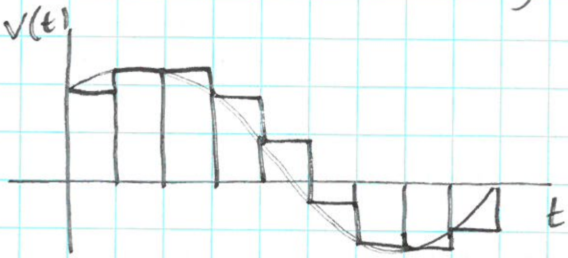
5.2 The Definite Integral.

LAST TIME ... We estimated distance traveled given velocity data or function.



using either left or right hand points.

more points = better estimate.

given $f(x)$ on $a < x < b$.

Left Hand Sum $= f(t_0)\Delta t + f(t_1)\Delta t + \cdots + f(t_{n-1})\Delta t$ $\qquad t_0 = a$

Right hand Sum $= f(t_1)\Delta t + f(t_2)\Delta t + \cdots f(t_n)\Delta t$ $\qquad t_n = b$

and $\Delta t = \dfrac{b-a}{n}$ $\qquad n = \#$ of subdivisions.

Accuracy of Estimates $= \left| \begin{array}{l} \text{Difference between} \\ \text{RHS and LHS} \end{array} \right| = |f(b) - f(a)|\Delta t.$

Another way to write these ...

$$\text{LHS} = \sum_{i=0}^{n-1} f(t_i)\Delta t \qquad \text{RHS} = \sum_{i=1}^{n} f(t_i)\Delta t$$

$\sum \Rightarrow$ add up the terms $\quad i=0$ starting point $\quad (n-1)$ ending point.

Last time our best estimate is when # of data points or # of divisions goes to ∞!

This is the limit! as $n \to \infty$!

THE DEFINITE INTEGRAL

Suppose f is continuous for $a \le x \le b$. The definite integral from a to b:

$$\int_a^b f(t)\,dt$$

is the limit of the LHS or RHS with n subdivisions as n gets large:

$$\left.\begin{aligned} \int_a^b f(t)\,dt &= \lim_{n\to\infty}\left(\sum_{i=0}^{n-1} f(t_i)\,\Delta t\right) \\ \int_a^b f(t)\,dt &= \lim_{n\to\infty}\left(\sum_{i=1}^{n} f(t_i)\,\Delta t\right) \end{aligned}\right\}$$

These limits are called Riemann Sums!

Worksheet In Class.

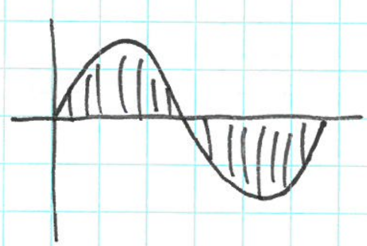
Quick Ideas:

More Generally: $\displaystyle\int_a^b f(t)\,dt = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta t_i$

sub divisions don't have to be equal.

This always represents the area under a curve!!

Guess $\displaystyle\int_0^{2\pi} \sin(x)\,dx$



$= 0$!