• Go over survey ... hand in homework...

Last time: we defined the definite integral

$$\int_a^b f(t)\,dt = \underbrace{\lim_{n\to\infty}\left(\sum_{i=0}^{n-1} f(t_i)\,\Delta t\right)}_{\text{Riemann sum.}}$$

left hand sum.

or right hand sum also works!

We found that the riemann sum does in fact give us the value of the integral as $n \to \infty$ spreadsheet

AND the integral is the area under the curve or between the curve an $x=0$.

Units ... $\int_a^b f(t)\,dt$ if $f(t)$ is velocity at time t in meters/second

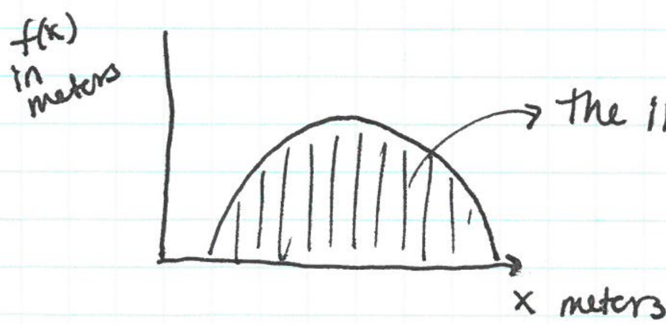
then we found that $\int_a^b f(t)\,dt$ = distance = meters.

we could think of this as $f(t) = \frac{\text{meters}}{\text{second}}$ dt = seconds

so $f(t)\,dt$ = meters.

Ex What are the units of $\int_a^b f(x)\,dx$ if $f(x)$ is meters and x is in meters.

then $\int_a^b f(x)\,dx$ = meters · meters = meters^2 Area



the integral gives the area under the curve.

The Fundamental Theorem of Calculus...

We saw that change in position can be calculated by the integral of velocity

$$\int_a^b f(t)\,dt = \text{change in position from } t=a \text{ and } t=b$$

What if we knew ^or could figure out. a function for position

$F(t) =$ position at time t

then $F(b) - F(a) =$ change in position from $t=a$ to $t=b$

but wait... how are position and velocity related?

$F'(t) = \text{velocity} = f(t)$

so
$$\boxed{\int_a^b f(t)\,dt = F(b) - F(a) \quad \text{where } f(t) = F'(t)}$$

so solving the integral comes down to figuring out $F(t)$ and evaluating it at the end points.

Applications of the Fundamental theorem...

let's first understand what this means!

Ex Let $F(t)$ represent a bacteria population where $F(0) = 5$ million. After t hours the population is growing at a rate of 2^t million bacteria/hour

Estimate the total increase in bacteria population during the first hour and the population at $t=1$.

The rate $F'(t) = 2^t = f(t)$

so the change in population between $t=0$ and $t=1$ $= F(1) - F(0) = \int_0^1 2^t \, dt$

or $F(1) = \int_0^1 2^t \, dt + F(0) = 1.44 + 5$

Think what is $\frac{d}{dt} 2^t = \ln(2) 2^t$

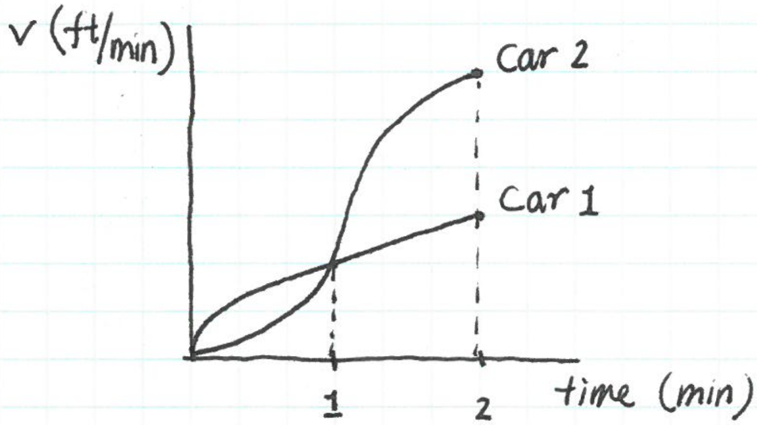
so going backward $F(t) = \frac{1}{\ln(2)} 2^t$ then $f(t) = 2^t$

for now use a calculator to evaluate

$F(1) = 6.44$ million bacteria

and the change in population $= 1.44$ milli bacte

<u>EX</u> Two cars start at rest at a traffic light and accelerate for several minutes



(a) Which car is ahead after 1 minute?

Car 1 is ahead it went faster...

(b) Which car is ahead after 2 min

compare $\int_0^2 V_1 \, dt$ and $\int_0^2 V_2 \, dt$ AREA UNDER THE CURVE!

Distance traveled in two minutes

Car 2 is ahead, more area under the curve $\Rightarrow$ it traveled further.

Copy Example 4 ... ~~Blow up Figure~~! Hand out worksheet.

~~and the table~~!

Ex The function $F(t)$ measures the # of calories eaten at thanksgiving dinner t minutes into the meal.

assume $f(t) = F'(t)$

1. What are the units of $f(t)$?
2. What does $f(4) = 50$ mean?
3. Could $f(t)$ ever be negative?
   What does this mean?
4. What does
   $\lim_{t \to \infty} F(t) = 2500$ tell us about our thanksgiving meal?
5. What are the units of $\int_0^{10} f(t)\,dt$?
   - What does this integral tell us?
6. What about $\int_0^{20} f(t)\,dt$?
7. If $F(0) = 0$, $F(10) = 1000$ and $F(20) = 2500$
   evaluate the following:
   $\int_0^{10} f(t)\,dt$ and $\int_0^{20} f(t)\,dt$.

BREAK INTO GROUPS ... ?? If time...

- Come up with your own function $F(t)$ examples complete with units.
- Describe the derivative and what it means. Is it positive negative or zero
- Talk about what $\int_a^b f(t)\,dt$ measures.

Calculus 1 - Meaning of the Definite Integral

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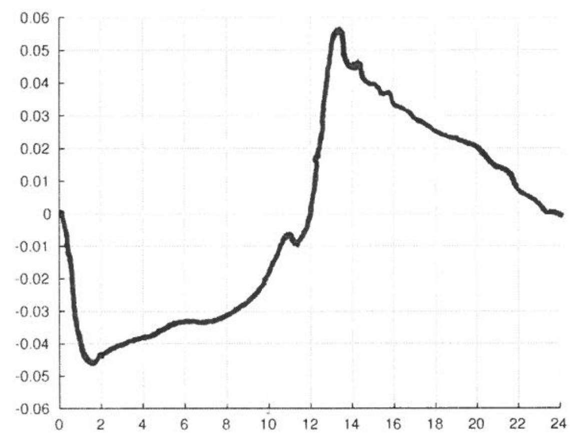
Example 4 from our textbook: Biological activity in water is reflected in the rate at which carbon dioxide, CO_2, is added or removed. Plants take CO_2 out of the water during the day for photosyntheses and put CO_2 into the water at night. Animals put CO_2 into the water all the time as they breathe. The figure shows the rate of change of the CO_2 level in a pond where t measures the time since dawn. At dawn there was 2.600 mmol of CO_2 per liter of water.



The following table gives the rate at which CO_2 is entering or leaving the water

t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$
0	0.000	4	-0.039	8	-0.026	12	0.000	16	0.035	20	0.020
2	-0.044	6	-0.035	10	-0.020	14	0.045	18	0.027	22	0.012

1. What are the units of $f(t)$? In words explain what would the following integral represent?

$$\int_0^3 f(t)\,dt$$

2. At what time was the CO_2 level the lowest? Highest?
3. Estimate how much CO_2 enters the pond during the night ($t = 12$ to $t = 24$).
4. Estimate the CO_2 level at dusk (twelve hours after dawn).
5. Does the CO_2 level appear to be approximately in equilibrium? (ie. after an entire day does the CO_2 remain about the same?)