DAY 3

Ch. 2.1 How do we measure speed?

DRAG RACE .25 mile track
cars start at 0 mph
by the finish pro's are > 300 mph.

We are spectators and want to know the speed of the cars. How can we measure this?

– TAKE SNAPSHOTS IN TIME

TIME	0	3	5	6	6.5	7
DISTANCE	0	.05	.1	.15	.2	.25

What was the speed in the first three seconds?

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{.05}{3} = .0166 \text{ miles/s} \cong 60 \text{ mp}$$

QUESTIONS: Was the car going 60 mph for the whole 3 seconds?
What was it doing?

SO REALLY we have defined an AVERAGE SPEED

Formula

$$\text{AVERAGE VELOCITY} = \frac{\text{Change in Position}}{\text{Change in Time}}$$

NOTE
velocity vs speed
velocity has a direction.

if $s(t)$ is the position at time t then the average velocity of an object over the interval $a \le t \le b$ is

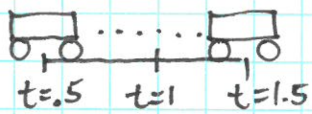
$$\text{AVERAGE VELOCITY} = \frac{s(b) - s(a)}{b - a}$$

What if instead I ask: HOW FAST WAS THE CAR GOING AT EXACTLY $t=1$

WHAT WAS THE INSTANTANEOUS VELOCITY ???
how can we measure this?

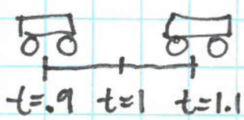
SNAPSHOT: 

looks like the car is going 0 mph

MOTION: 

OK... but this is still an average between $.5 \le t \le 1.5$

How can we do better... ZOOM IN

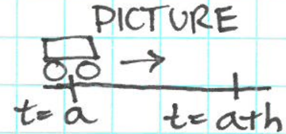


Better but still an Average

Lets say we keep getting closer and closer to $t=1$... approaching $t=1$ from both sides
What does this sound like mathematically?

TAKING A LIMIT !! Thanks to Calculus and the Idea of a limit we can define instantaneous velocity.

Rewrite
average velocity $= \dfrac{s(a+h)-s(a)}{h}$

PICTURE


distance: $s(a+h)-s(a)$
time: $a+h-a=h$

THEN INSTANTANEOUS VELOCITY:

$$\lim_{h\to 0} \frac{s(a+h)-s(a)}{h}$$

let $s(t)$ be the position at time t then the instantaneous velocity at time $t=1$ is

Graphically.

ONE MORE IDEA.... LETS GRAPH THE POSITION OF THE CAR OVER TIME.

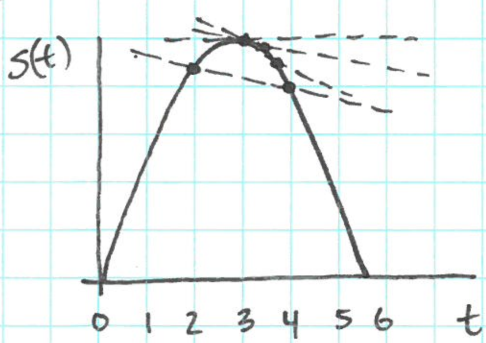
describe the motion of the car...

starts @ 0 mph and speeds up more and more until it reaches it's top speed.

AVERAGE VELOCITY: slope of a line connecting two points on a curve

INSTANTANEOUS VELOCITY: slope of a line at a single point.

EX Consider a more drastic example: s(t) is the distance vertically from the surface of the earth.



Describe this motion...

like throwing a baseball straight up and then watching it fall back down.

AVERAGE VELOCITY $2 \le t \le 4$: $\text{Average Velocity} = \dfrac{s(4) - s(2)}{4 - 2}$

INSTANTANEOUS VELOCITY @ $t = 3$

$$\lim_{h \to 0} \frac{s(3+h) - s(3)}{h}$$

— this is a tangent line.

[redacted] graph

.1

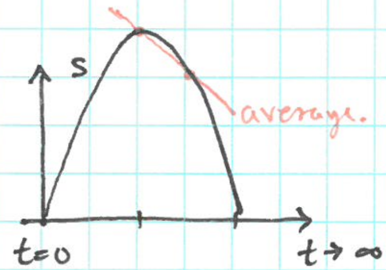
[redacted] approaching.

Try this for a concrete example ...

$$s(t) = -(t-2)^2 + 4$$

s = meters

t = minutes



Find the average velocity between $t=2$ and $t=3$

$$\text{Average Velocity} = \frac{s(3)-s(2)}{3-2} = \frac{3-4}{1} = -1 \text{ m/min.}$$ UNITS.

graph this

Find the instantaneous velocity.... at $t=3$

$$V = \text{Instantaneous Velocity} = \lim_{h\to 0} \frac{s(3+h)-s(3)}{h} = \lim_{h\to 0} \frac{-(3+h-2)^2+4-3}{h}$$

$$= \lim_{h\to 0} \frac{-(1+h)^2+1}{h}$$

TWO WAYS: NUMERICALLY ... choose smaller and smaller h.
ALGEBRAICALLY ... find the limit exactly.

<u>Numerically</u> we are SNEAKING UP ON $h \to 0$ choose smaller and smaller values...

$h = -.1$ $\quad V \approx -1.9$ $\quad$ LEFT — $\lim_{h\to 0^-}$

$h = -.01$ $\quad V \approx -1.99$

$h = -.001$ $\quad V \approx -1.999$ ... what # are we approaching?

$h = .1$ $\quad V \approx -2.1$ $\quad$ RIGHT — $\lim_{h\to 0^+}$

$h = .01$ $\quad V \approx -2.01$

$h = .001$ $\quad V \approx -2.001$ ... what # are we approaching?

In both cases we approach -2!

$$\lim_{h\to 0} \frac{-(1+h)^2+1}{h} = -2$$

Algebraically...

$$\lim_{h\to 0} \frac{-(1+h)^2+1}{h} = \lim_{h\to 0} \frac{-h^2-2h-1+1}{h}$$

$$= \lim_{h\to 0} -h-2 = -2 \quad \checkmark \quad \text{matches.}$$

YOU TRY... $s(t) = t^2$ want velocity @ $t=3$.