6.1 Antiderivatives

DEFINITION

If the derivative of F is f then we call F the antiderivative of f

Ex. What is the antiderivative of $2x$?

x^2 , x^2+1 , x^2+2 , ... $x^2 + c$

(wouldn't these work too)

we lose constants when we take a derivative.

Ex Say we know the velocity of a car, $v(t)$. The position of the car is the antiderivative:

$$\frac{ds}{dt} = v(t)$$

$\frac{ds}{dt} = 100$

$v(t) = 100$ mph where am I located after one hour?

* depends on where you started

* I am 100 miles from where I started

Antiderivative of $s(t) = 100$ is $S(t) = 100t + C$

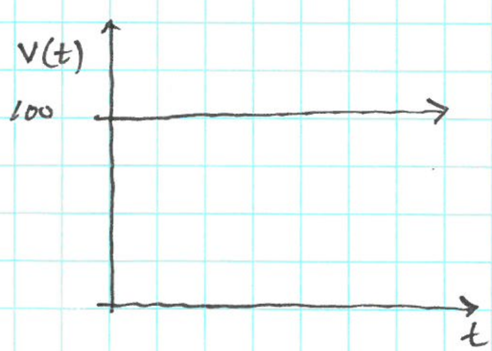
$s(t) = 100t + C$ → the unknown constant tells us where we started.

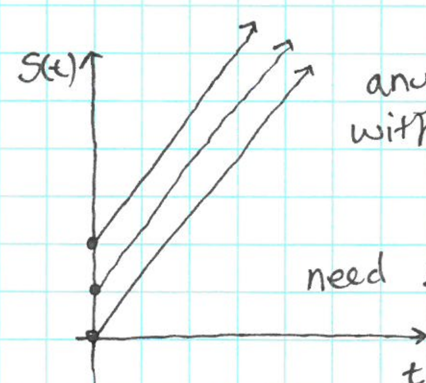
GRAPHICALLY

$s(0) = c$!!

INITIAL CONDITION.

what does this look like on a graph...





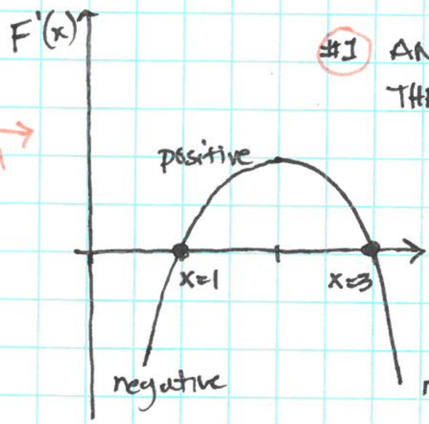
any line with slope 100!!

need $s(0)$ to choose.

GENERAL ANTIDERIVATIVES.

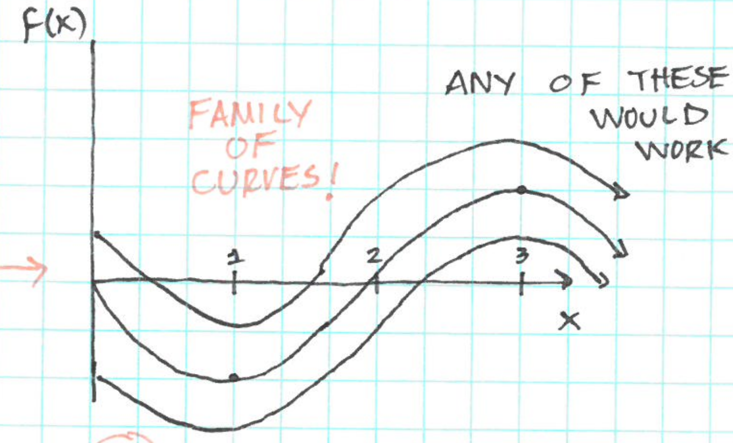
EX Draw curves that work as antiderivatives for $F'(x)$

GIVEN GRAPH →



#1 ANALYZE THE GRAPH OF $F'(x)$

DRAW →



CRITICAL POINTS: $x=1$ $x=3$

MAX OR MIN: $\vee$ MIN $\wedge$ MAX

• First derivative test

#2 DRAW MAX OR MINS

#3 CHOOSE A STARTING POINT

#4 DRAW CURVE BASED ON SLOPE.

Some important Ideas:

THE INTEGRAL → $\int_a^b f(t)\,dt$ Finds the area under the curve.

If $f(t)$ is velocity (m/s) then $\int_a^b f(t)\,dt$ finds the distance traveled between $t=a$ and $t=b$

Given $\int_a^b f(t)\,dt$ a and b are the LIMITS of INTEGRATION
$f(t)$ is the INTEGRAND

If $f(t) = F'(t)$ then we call $F(t)$ the ANTIDERIVATIVE.
this is a FAMILY of CURVES

FUNDAMENTAL THEOREM $\int_a^b f(t)\,dt = F(b) - F(a)$ What happened to the constant here?

Well if $F(t) = x^2 + c$ then $F(b) - F(a) = (b^2+c) - (a^2+c)$
the constant cancels! $= b^2 - a^2$

6.2 Constructing Antiderivatives Analytically

- for many cases... given $f(x)$ we can figure out $F(x)$!!
- If we can't find $F(x)$ then we go back to evaluating LEFT AND RIGHT sums or AREA UNDER CURVE.
- As a lazy mathematician I would rather get good at finding antiderivatives!

TWO TYPES OF INTEGRALS

more general

$$\int f(x)dx = F(x) + C$$

$\boxed{\text{INDEFINITE}}$

don't know a and b

ANSWER IS A FUNCTION + CONSTANT OR A FAMILY OF FUNCTIONS

more specific... I have start and end points.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$\boxed{\text{DEFINITE}}$

we know a and b

ANSWER IS A NUMBER

USE THE FUNDAMENTAL THEOREM

in general we use the indefinite integral

the definite integral is used when we are finding a # or soln to a problem.

HERE WE GO ... evaluate the following:

ARE THESE DEFINITE or INDEFINITE ?

$$\int 1\,dx = \int dx = x + c$$

$$\int 2x\,dx = x^2 + c$$

$$\int 3x^2\,dx = x^3 + c$$

these ones are straight forward because they are exact derivatives.

$$\int x^2\,dx = \frac{1}{3}x^3 + c$$

A harder one!

more generally I have to account for the constant factor.

How do I do this?

$$\int x^2\,dx = \frac{1/3}{}\,x^3$$

BUT $\frac{d}{dx}x^3 = 3x$

I must balance the 3 with a 1/3

I know x^2 comes from derivative of x^3

How can I check? TAKE THE DERIVATIVE $\frac{d}{dx}\frac{1}{3}x^3 = x^2$ this MUST match my integrand.

YOU TRY:

$$\int x\,dx = \frac{1}{2}x^2 + c$$

$$\int 3x\,dx = \frac{3}{2}x^2 + c$$

constant multiple rule or just see the constant should cancel out.

$$\int 2x^4\,dx = \frac{2}{5}x^5 + c$$

CAN ANYONE SEE A PATTERN?

GENERAL RULES.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + c$$

need the absolute value because 1/x undefined at x=0

$$\int \frac{1}{\text{cabin}}\,d\,\text{cabin} = \ln|\text{cabin}|$$

:)

$$\int e^x \, dx = e^x + c \qquad \int \cos x \, dx = \sin x + c$$

$$\int a^x \, dx = \frac{a^x}{\ln(a)} + c \qquad \int \sin x \, dx = \cos x + c$$

NOW WE CAN START TO INTEGRATE MORE COMPLICATED FUNCTIONS.

- in general, integration takes practice
  it is harder to go backward
  than forward.

Ex $\int 3x + x^2 \, dx$ use properties of integration

$$= \int 3x\,dx + \int x^2\,dx = \frac{3}{2}x^2 + \frac{1}{3}x^3 + c$$

Ex $\int \sin x + 3\cos x \, dx$

$$= -\cos x + 3\sin x + c$$

Ex $$\int_1^2 3x^2 \, dx = x^3 \Big|_1^2 = (2)^3 - (1)^3 = 8 - 1 = 7.$$

don't need constants
in the definite case!

Ex $$\int_{-1}^{1} x^2 + x^3 \, dx = 2\int_0^1 x^2 \, dx = \frac{2}{3}x^3 \Big|_0^1 = \frac{2}{3}$$

HOW CAN I CHECK MY ANSWER?

WHAT IF I RUN INTO A FUNCTION
THAT I CANT INTEGRATE?