Last time – Antiderivatives
Definite Integrals
Indefinite Integrals.
} Define these!

Today: Differential Equations:

Any equation that relates a derivative to other derivatives or functions.

Ex $\frac{ds}{dt} = t^2 + 1$ $s(t)$ = position in m at time t seconds.

To solve ... use antiderivatives or Integrals:

$$\int \frac{ds}{dt}\,dt = \int t^2 + 1\,dt$$ integrate both sides.

$$s(t) = \frac{t^3}{3} + t + C$$ don't forget the constant.

↑ to know the position we need an initial condition.

SAY $s(0) = 1$

$$s(0) = \frac{0^3}{3} + 0 + C = C = 1 \quad \text{so} \quad C = 1$$

Use the initial condition to find the constant

ANSWER $\boxed{s(t) = \frac{t^3}{3} + t + 1}$

Differential equations are used a ton in Physics and Engineering:

$s(t)$ = position

$\frac{ds}{dt}$ = velocity = $v(t)$

$\frac{dv}{dt}$ = acceleration = $a(t) = \frac{d^2s}{dt^2}$

} knowing one we can solve for the others!

If dealing with gravity we use $g = 9.8\ m/s^2 = 32\ ft/s^2$ ACCELERATION

Ex A stone is dropped from a 100 m building. Find functions for position and velocity. How fast is it going when it hits the ground?

$a(t) = -9.8\ m/s^2$ acceleration from gravity.

#1 Write down the problem mathematically. Relate velocity, position etc..

$\frac{dv}{dt} = -9.8 \quad v(0) = 0$ "DROPPED"

#2 Solve by integration. $\int \frac{dv}{dt}\,dt = \int -9.8\,dt$

$v(t) = -9.8t + C$ integration

$v(0) = C = 0$ initial condition

so $\boxed{v(t) = -9.8t}$

But we need position too! $\frac{ds}{dt} = v(t) = -9.8t$

$s(0) = 100$ start at TOP.

$\int \frac{ds}{dt}\,dt = \int -9.8t\,dt$

$s(t) = -\frac{9.8t^2}{2} + C \qquad s(0) = C = 100$

$$\boxed{s(t) = -\frac{9.8t^2}{2} + 100}$$

Now answer questions ...

Hits ground $\Rightarrow s = 0 \qquad -\frac{9.8t^{*2}}{2} + 100 = 0$

$t^{*2} = \frac{200}{9.8} \qquad t^* = \pm\sqrt{\frac{200}{9.8}}$

$$\boxed{t^* = 4.518\ s.}$$

HOW FAST $v(4.518) = -9.8(4.518)$

$$\boxed{v(t^*) = -44.272\ m/s.}$$

A car is driving at 50 mph. It is driving away from us and started 10 miles away. What is the position function?

velocity = 50 mph

$\frac{ds}{dt} = 50$ differential eqn.

$s(0) = 10$ initial condition

:m
xtinally.

the differential
n by integration

$$\int \frac{ds}{dt}\,dt = \int 50\,dt$$

$$s(t) = 50t + C$$

he initial
n to find
e constant

$$s(0) = 50(0) + C = C = 10$$

$$\boxed{s(t) = 50(t) + 10}$$

Ex Your awesome professor has sealed the exam solutions in a plastic container and plans to drop them from the top of appleton hall. She does not throw them so the initial velocity is zero and the initial position is the top of the building, minus about 5 feet, your height. v_0 30 feet.

Recall the acceleration of gravity is $-32\ ft/s^2$ when do the answers hit the ground?

1. Write down the problem. $a = -32$ $\quad \frac{dv}{dt} = -32$ or $\frac{d^2s}{dt^2} = v(t)$

$v(0) = 0$ and $s(0) = 30 + h$

2. Solve by integration TWO STEPS

STEP ONE

$\frac{dv}{dt} = -32 \qquad \int \frac{dv}{dt}\,dt = \int -32\,dt$

$v(t) = -32t + c$

Apply velocity condition: $v(0) = 32(0) + c = c = 0$

$c = 0$

$v(t) = -32t$

STEP TWO

$\frac{ds}{dt} = -32t \qquad \int \frac{ds}{dt}\,dt = -\int 32t\,dt$

$s(t) = -16t^2 + c$

$s(0) = -16(0) + c = c = 30 + h$

$s(t) = -16t^2 + 30 + h$

Now answer the question ... when do they hit the ground ... when $s = 0$ and $h = 0$

$-16t^2 + 30 = 0 \qquad t^2 = 30/16 = 1.875\ s^2 \qquad t = \pm\sqrt{2.1875}\ s$

$t = 1.389\ s$

Plug in Your height ... when should you be there.

A helicopter is hovering at 6400 feet and drops a bag containing 1 million dollars. At what time will the bag hit the ground?

$\frac{dv}{dt} = -32 \qquad v(0) = 0$

$v(t) = -32t + C \qquad C = 0 \qquad v(t) = -32t$

$\frac{ds}{dt} = -32t \qquad s(t) = -16t^2 + C$

$s(0) = -16(0) + C = 6400$

$s(t) = -16t^2 + 6400$

Hits the ground when $s = 0 \qquad -16t + 6400 = 0$

$t^2 = \frac{6400}{16} = 400$ s.

$t = \pm 20$

1 mile away
5280

You are driving a convertable sports car and are ~~2400~~ 5280 feet away. How fast do you need to be driving to catch the money.
[HINT → let your constant velocity $V = V_0$]

$\frac{ds}{dt} = V_0 \qquad s(t) = V_0 t + C$

$s(0) = -1200 \qquad s(0) = V_0(0) + C = -5280$

$C = -5280$

$s(t) = V_0 t - 5280$

Want $s(20) = 0$

$V_0(20) - 5280 = 0$

$V_0 = \frac{5280}{20} = \frac{120}{2} = 264$ ft/s.

$\sim$ 180 mph

Ex A Boston Creme doughnut is thrown vertically upward with a speed of 10 m/s from a height of 2 meters. What is the highest point it reaches? When does it hit the ground?

1. Write down the equations

$$\frac{dv}{dt} = -9.8$$
$$v(0) = 10$$
$$s(0) = 2$$

2. Solve for position and velocity.

$$v = -9.8t + C$$
$$v(0) = C = 10$$
$$\boxed{v(t) = -9.8t + 10}$$

$$\frac{ds}{dt} = -9.8t + 10$$
$$s(t) = -4.9t^2 + 10t + C$$
$$s(0) = C = 2$$
$$\boxed{s(t) = -4.9t^2 + 10t + 2}$$

3. Now answer the questions.

Highest point = maximum position!

$s'(t) = 0$ OR $v(t) = 0$

$$-9.8t + 10 = 0$$
$$t = 10/9.8 = 1.02 \text{ s.}$$

Hits the ground when $s(t) = 0$

$$-4.9t^2 + 10t + 2 = 0$$

how do we solve this?
quadratic formula

$t = -0.18$ and 2.22

$t = 2.22$ seconds it hits the ground.

General Solutions to Differential Eqns:

$$\frac{dy}{dx} = f(x)$$

- to find the soln ... find the antiderivative of $f(x)$ ... basically just integrate.

Ex $\frac{dy}{dx} = \sin x + 2$ solve the differential eqn...

$$y = \int \sin x + 2 \, dx = -\cos x + 2x + C$$

must add the constant
soln is a family of curves.

How do we solve for C?
we are given additional info!

$y(0) = 0$ plug this in!

$$y(0) = -\cos(0) + 2(0) + C = -1 + C = 0$$
$$\text{so } C = 1$$

with this condition $\boxed{y(x) = -\cos x + 2x + 1}$

This is called an initial value problem!

Ex. Find the solution

$$\frac{dy}{dx} = \sin x + 2, \quad y(3) = 5$$

Ex Find the soln

$$\frac{dP}{dt} = 10e^t, \quad P(0) = 25.$$