DAY 4

Recall:

$s(t) \Rightarrow$ position at time t

$$\text{Average Speed} = \frac{s(a+h) - s(a)}{h} \quad \text{-OR-} \quad \frac{s(b) - s(a)}{b-a}$$

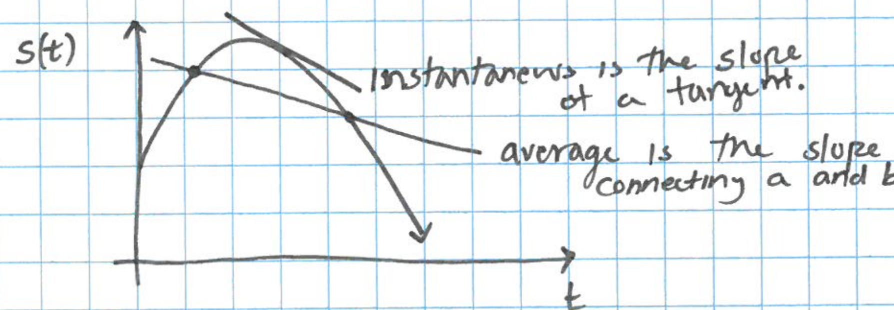
$$\text{Instantaneous Speed} = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$

we can calculate this limit TWO WAYS

Numerically – plug in smaller and smaller values of h ... from both sides.

Algebraically – just evaluate the limit.

GRAPHICALLY



Lets go through one more example...

$s(t) = t^3 + 3$ meters

$t =$ seconds

$$\text{Average speed between } t=0 \text{ and } t=1 = \frac{s(1) - s(0)}{1-0} = \frac{4-3}{1} = 1 \text{ meters/second.}$$

$$\text{Instantaneous Speed at } t=1 = \lim_{h \to 0} \frac{s(1+h) - s(1)}{h} = \lim_{h \to 0} \frac{[(1+h)^3 + 3] - [4]}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^3 - 1}{h}$$

TWO WAYS ...

Numerically ... plug in.

$h = -.1$ est $= 2.71$
$h = -.01$ $= 2.97$
$h = -.001$ $= 2.997$ seem to go to $\underline{3}$

$h = .001$ $= 3.003$
$h = .01$ $= 3.03$ ✓ matches.
$h = .1$ $= 3.31$

so $\boxed{\lim_{h\to 0} \frac{(1+h)^3 - 1}{h} \sim 3}$

Algebraically.

$$\lim_{h\to 0} \frac{(1+h)(1+2h+h^2)-1}{h} = \lim_{h\to 0} \frac{(1+2h+h^2+h+2h^2+h^3)-1}{h}$$

$$= \lim_{h\to 0} \frac{h^3+3h^2+3h+1-1}{h} = \lim_{h\to 0} h^2+3h+3 = 3$$

$$\boxed{\text{Instantaneous Speed} = 3 = \underline{\text{slope}} \text{ of The tangent line}}$$

Eqn of tangentline at $t=1$?

- we know the slope $m=3$
  $y = 3x + b$
- plug in a point $s(0) = 3$ $x=0$ $s=3$ $(0,3)$
  $3 = 3(0) + b$ $b=3$

TANGENT LINE $\boxed{y = 3x + 3}$

BUT we could apply this to ANY function not just position!

The instantaneous rate of change $= \text{DERIVATIVE} = f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

What if $f(x)$ represented # of hairballs my cat barfs up at time x (seconds) after feeding.

$f(0) \longrightarrow$ # of hairballs zero seconds after feeding.

$f(1) = 10$ what does this mean?

$f'(1) = 2$ what does this mean? NOT GOOD!!

Independent variable is not always TIME.

What if $f(x)$ represents my ~~[illegible]~~ Energy level while riding my bike to school and x is miles from home.

$f(0) = 100$
$f'(1) = 2$
$f'(3) = -10$

what do these mean?