Day 5

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2.4 Interpreting The derivative $\longrightarrow$ Next time 2.3...

given $y = f(x)$

$$\text{The Derivative} = f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

this measures: $\dfrac{\text{difference in } y}{\text{difference in } x.}$

If $y = f(x)$ is ...

DISTANCE

$s = f(t)$ s = position, t = time.

then $\frac{ds}{dt} = f'(t) =$ speed

change in position over time.

$s(1) = 10$

$s'(1) = -1$ At The moment $t=1$ position is decreasing.

TEMPERATURE

$T = f(t)$ T = temp, t = time

then $\frac{dT}{dt} = f'(t) =$ change in Temp.

change in temperature over time.

$T(1) = 10$

$T'(1) = 3$ At The moment $t=1$ the temperature is increasing.

Estimate $T(2)$

well $T(1) = 10$ and it is changing 3 per step so...

$T(2) \sim 13$

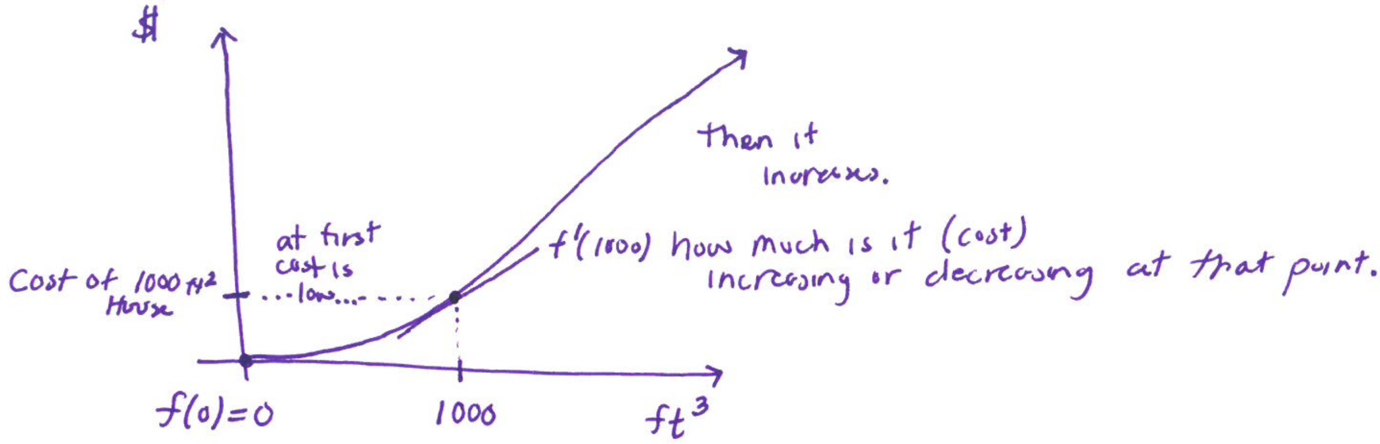
Lets do lots of examples!

Ex $C = f(A)$ cost (\$) of building a house is dependent of The area in square feet.

What does $\frac{dC}{dA}$ mean? UNITS $\dfrac{\text{change in cost (dollars)}}{\text{change in area (sq feet)}}$

$f'(1000) = 10$ ... I have 1000 square feet if I went up to 1001 it would cost about \$10 more.

What might this function look like...



Ex $N = f(c)$ Number of students who eat at the commons is dependent on the cost of food \$.

what should $N = f(0) = ?$ LOTS! $= 2000$
It's FREE!!

What does N do as C increases?

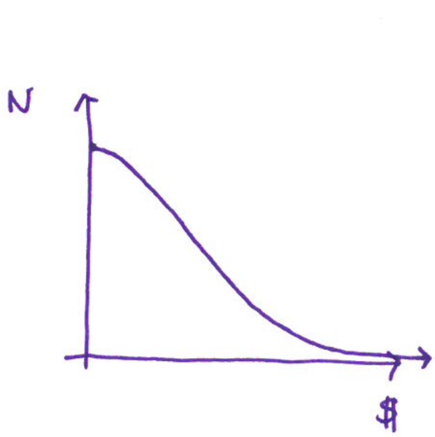
What would $N' = f'(10) = -20$ expect pos or ~~neg~~?

<u>UNITS</u> $\left(\dfrac{\text{Students}}{\$}\right)$ fewer students would eat if we increased the cost.

If $f(10) = 1000$ what might we expect

$f(10.5) = \cancel{1001}$ 990 students

$f(9.5) = 1010$ students.



what might we expect

$\lim_{\$\to\infty} f(c) = 0$?

$\lim_{\$\to\infty} f'(c) = 0$?

Ex $E = f(d)$ Is the effectiveness of a drug given a dose (ml)

effectiveness — measured in % of population that recovers in 3 days.

What does $f(1) = 10$ mean?

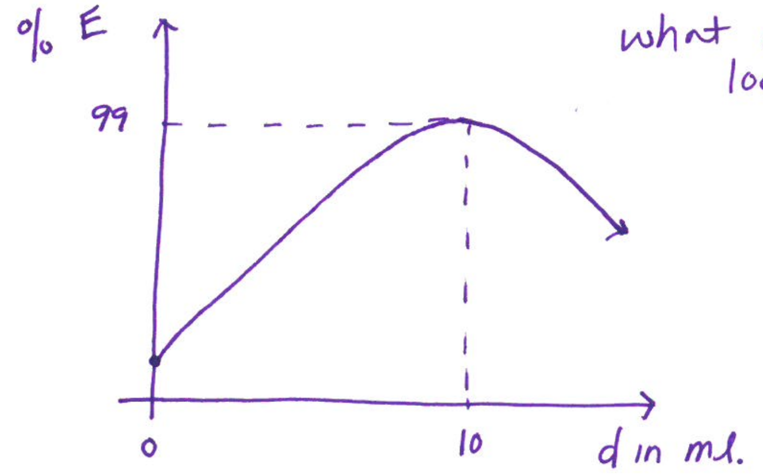
What do we think $f(0) = \underset{?}{.5}$ should be?

depends ... maybe some people recover w/o drug
but if the drug works at all it is less than 10.

If $f'(1) = 10$ should we give more of the drug?
Yes — the effectiveness is increasing !!

If $f(10) = 99$ and $f'(10) = -1$ what does this tell us?

Giving a dose of 10 ml is 99% effective
but giving more decreases effectiveness.
So we should <u>not</u> give more.



what might this function look like...

Ex What if I can tell you exactly the change in stock market prices per day... into the FUTURE ☺

$S = f'(d) \longrightarrow$ I know the rate of change...

what would $f(d)$ tell me?

$f(d)$ — the stock market price on date d.

Work to get use to thinking about what functions mean. This is the best way to actually be able to apply Calculus to real world problems!!

Section 2.4 – Interpretations of the Derivative

1. Let $f(p)$ represent the daily demand for San Francisco '49ers T-shirts when the price for a shirt is p dollars. In other words, $f(p)$ gives the number of shirts purchased daily if the selling price is p dollars.

   (a) Is f increasing or decreasing?

   (b) What are the units of p, $f(p)$, and $f'(p)$?

   (c) Explain, in terms of shirts and dollars, the practical meaning of the following:

      i. $f(20) = 150$

      ii. $f'(20) = -5$

      iii. $f(30)$

   (d) Let d represent demand. Then $d = f(p)$, so the function f takes ________ as an input and gives ________ as an output. On the other hand, the inverse function f^{-1} takes ________ as an input and gives ________ as an output, so $f^{-1}($____$) =$ ____.

   (e) Give practical interpretations of $f(25)$ and $f^{-1}(25)$.

2. (Taken from *Hughes-Hallett, et. al.*) If t is the number of years since 1993, the population, P, of China, in billions, can be approximated by the function

$$P = f(t) = 1.15(1.014)^t.$$

   (a) Calculate and interpret $f(6)$ in the context of this problem.

   (b) Use the table method to estimate $\frac{dP}{dt}$ at $t = 6$, and give an interpretation of this number in the context of this problem.

3. Between noon and 6 p.m., the temperature in a town rises continually, but rises at its quickest around 3 p.m., and slowest around noon and 6 p.m.

   (a) Sketch a possible graph of $H = f(t)$, where H is the temperature in the town (in degrees Fahrenheit) and t represents the time (in hours) after 12:00 noon.

   (b) Explain, in terms of degrees and hours, what each of the following represents:

      (i) $f'(2)$ (ii) $f'(3) = 7$ $f(4) = 40$ $f'(4) = 1$

   (c) Use the statements given in parts (iii) and (iv) from above to estimate the temperature in the town at 5:30 p.m. Is the actual temperature higher or lower than the estimate?

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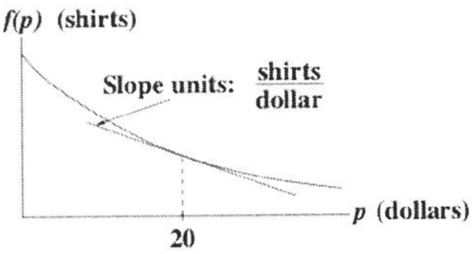
1. Let $f(p)$ represent the daily demand for San Francisco '49ers T-shirts when the price for a shirt is p dollars. In other words, $f(p)$ gives the number of shirts purchased daily if the selling price is p dollars.

   (a) Is f increasing or decreasing?

   f is decreasing.

   (b) What are the units of p, $f(p)$, and $f'(p)$?

   p has units of dollars, and $f(p)$ has units of shirts. Since $f'(p)$ represents the rate of change of $f(p)$ with respect to p, its units are shirts per dollar.

   (c) Explain, in terms of shirts and dollars, the practical meaning of the following:

   i. $f(20) = 150$

   This states that if the price for a shirt is \$20, then 150 shirts will be bought daily.

   ii. $f'(20) = -5$

   This states that if the price for a shirt is \$20, then the daily demand for shirts will decrease by about 5 shirts for each additional dollar that is charged for the shirt.

   iii. $f(30)$

   This represents the number of shirts that will be bought in a day if the price for a shirt is \$30.

   (d) Let d represent demand. Then $d = f(p)$, so the function f takes price as an input and gives demand as an output. On the other hand, the inverse function f^{-1} takes demand as an input and gives price as an output, so $f^{-1}(\underline{d}) = \underline{p}$.

   (e) Give practical interpretations of $f(25)$ and $f^{-1}(25)$.

   $f(25)$ represents the number of shirts bought daily if the selling price is \$25.

   $f^{-1}(25)$ represents the selling price of a shirt that would result in 25 shirts being bought daily.

2. (Taken from *Hughes-Hallett, et. al.*) If t is the number of years since 1993, the population, P, of China, in billions, can be approximated by the function

$$P = f(t) = 1.15(1.014)^t.$$

   (a) Calculate and interpret $f(6)$ in the context of this problem.

   $f(6) = 1.25$, which indicates that the population of China was 1.25 billion in 1999.

   (b) Use the table method to estimate $\frac{dP}{dt}$ at $t = 6$, and give an interpretation of this number in the context of this problem.

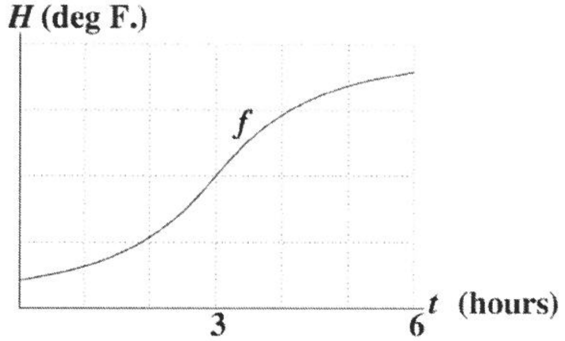
$$P'(6) = \lim_{h\to 0} \frac{1.15(1.014)^{6+h} - 1.15(1.014)^6}{h}$$

h	0.1	0.01	0.001
$\frac{1.15(1.014)^{6+h} - 1.15(1.014)^6}{h}$	0.01739	0.01738	0.01738

   From the calculations to the left, $P'(6)$ equals about 0.01738, which means that the population of China is increasing by about 17.4 million people per year in 1999.

3. Between noon and 6 p.m., the temperature in a town rises continually, but rises at its quickest around 3 p.m., and slowest around noon and 6 p.m.

(a) Sketch a possible graph of $H = f(t)$, where H is the temperature in the town (in degrees Fahrenheit) and t represents the time (in hours) after 12:00 noon.



(b) Explain, in terms of degrees and hours, what each of the following represents:

i. $f'(2)$

This represents the rate at which the temperature increases, in degrees per hour, at 2 p.m.

ii. $f'(3) = 7$

This states that at 3 p.m., the temperature is increasing at a rate of 7 degrees Fahrenheit per hour.

iii. $f(4) = 40$

This states that at 4 p.m., the temperature in the town is 40 degrees Fahrenheit.

iv. $f'(4) = 1$

This states that at 4 p.m., the temperature in the town is increasing at a rate of 1 degree Fahrenheit per hour.

(c) Use the statements given in parts (iii) and (iv) from above to estimate the temperature in the town at 5:30 p.m. Is the actual temperature higher or lower than the estimate?

$$\text{Estimated Temperature at 5:30 p.m.} = 40°\text{F} + (1\frac{°\text{F}}{\text{hr}})(1.5 \text{ hours}) = 41.5°\text{F}$$

This estimate is higher than the actual temperature at 5:30 p.m. because this calculation assumes that the temperature continues to increase at the same rate of 1 degree Fahrenheit per hour between 4 and 5:30 p.m. We can see from our graph in part (a), however, that f gets less and less steep between 4 and 5:30 p.m.