RULES FROM LAST TIME

Constant Multiple $\frac{d}{dx}\left[cf(x)\right] = c\frac{df}{dx} = cf'(x)$

Sum and Difference $\frac{d}{dx}\left[f(x)+g(x)\right] = f'(x) + g'(x)$

Power Rule $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Today... we start talking about more complicated functions and rules.

Exponential function. $g(x) = a^x$ $a =$ constant.

consider the limit

$$\frac{dg}{dx} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h\to 0} \frac{a^x a^h - a^x}{h}$$

$$= \lim_{h\to 0}\left(\frac{a^h - 1}{h}\right)a^x$$

so everything depends on this part...

NOTE... This is some sort of constant, it does not contain x.

So we can write:

$$\frac{d}{dx}\left(a^x\right) = k\,a^x$$

the derivative of a^x is just a constant times a^x.

now we need to investigate... what is this constant?

Start by asking: Is there an a such that the function is its own derivative?

MAGIC !!

In otherwords:

$$k = \lim_{h \to 0} \left(\frac{a^{*h} - 1}{h} \right) = 1 = \lim_{h \to 0} (1)$$

For super small values of h...

$$\frac{a^{*h} - 1}{h} \approx 1$$ use algebra to solve for a...

$$a^{*h} - 1 \approx h \qquad a^{*h} \approx h + 1 \qquad a^{*} \approx (h+1)^{1/h}$$

USING A CALCULATOR

If we were to sub in smaller and smaller values of h we would find.

$\lim_{h \to 0} (a) = \lim_{h \to 0} \left((h+1)^{1/h} \right)$

$h = .1$ $a \to 2.5937$

$h = .01$ 2.7048

$h = .001$ 2.7169

$h = .0001$ $a \to 2.7181$

$a^{*} \approx 2.718$ this is Eulers # e.

so

$$\lim_{h \to 0} \left(\frac{e^{h} - 1}{h} \right) = 1$$ (☆) We will use this again very soon.

AND

☆ $$\boxed{\frac{d}{dx} e^{x} = e^{x}}$$

$y = e^{x}$ is its own derivative!

RECALL our original formula

$$\frac{d}{dx} a^{x} = k a^{x}$$

we can use the above rule to find out what k is for any value of a.

I want to isolate constants and h's

→ I know about e.

The e is what is hard

MESS

$$k = \lim_{h \to 0} \left(\frac{a^{h} - 1}{h} \right) = \lim_{h \to 0} \left(\frac{(e^{\ln(a)})^{h} - 1}{h} \right) = \lim_{h \to 0} \frac{e^{\ln(a) h} - 1}{h}$$

So...

substitutions like this can really simplify things!!

A trick let $t = (\ln(a) h) \Rightarrow h = t / \ln(a)$

sub this in

as $h \to 0$ also $t \to 0$ → make sure to sub through everything.

$$\lim_{t\to 0} \frac{e^t - 1}{t/\ln(a)} = \lim_{t\to 0} \ln(a) \underbrace{\frac{e^t-1}{t}}$$

but we just said that as $t \to 0$ this piece goes to 1. FROM (★)

so $k = \lim_{t\to 0} \ln(a) \frac{e^t - 1}{t} = \ln(a)$

AND

$$\boxed{\frac{d}{dx} a^x = \ln(a)\, a^x}$$

Lets do some examples...

Ex $f(x) = 4e^x - 6^x$ find $f'(x)$

1. simplify if possible
   - This is already simplified
2. Use the rules to take the derivative.

$$\frac{df}{dx} = \frac{d}{dx}\left(4e^x - 6^x\right)$$

$$= \frac{d}{dx}\left(4e^x\right) - \frac{d}{dx}6^x \qquad \text{sum and Difference}$$

$$= 4\frac{d}{dx}e^x - \frac{d}{dx}6^x \qquad \text{constant mult.}$$

$$= 4e^x - \ln(6)6^x \qquad \text{exponential function rules.}$$

ok to leave it in this form.

What would $f''(x)$ be?

$$f''(x) = \frac{d}{dx}\left(4e^x - \ln(6)6^x\right) \qquad \text{you try...}$$

$$= 4e^x - [\ln(6)]^2 6^x$$

Guesses ... What is $f^{(10)}(x)$?

Ex $f(x) = 3e^{x+3} + 10x^2 - \sqrt{x}$

1. simplify if possible

$$f(x) = 3e^x e^3 + 10x^2 - x^{1/2}$$

These still follow the good old power rule.

write like this so my rule applies.

2. Use rules to take the derivative.

$$\frac{d}{dx}\left(3e^x e^3\right) + \frac{d}{dx}\left(10x^2\right) - \frac{d}{dx}x^{1/2}$$ sum/difference

$$3e^3\frac{d}{dx}e^x + 10\frac{d}{dx}x^2 - \frac{d}{dx}x^{1/2}$$ constant mult.

$$3e^3e^x + 20x - \tfrac{1}{2}x^{-1/2}$$ power and exponent rules.

3. Simplify.

$$f'(x) = 3e^{x+3} + 20x - \frac{1}{2\sqrt{x}}$$

Ex $f(x) = xe^x + 3e^{3x}$ Why can't we do this one yet?

PRODUCT RULE

CHAIN RULE

FOR NEXT TIME!

DO Worksheet IN CLASS