Problem 1 (0 points)

Evaluate the following integrals

$$\int \frac{1+z^2}{z^6}\,dz = \int \frac{1}{z^6}\,dz + \int \frac{1}{z^4}\,dz = \int z^{-6}\,dz + \int z^{-4}\,dz$$

$$= \left[-\frac{1}{5}z^{-5}\right] + \left[-\frac{1}{3}z^{-3}\right] + C = \frac{-1}{5z^5} - \frac{1}{3z^3} + C$$

$$\int \frac{1}{x} + 3e^x\,dx = \int \frac{1}{x}\,dx + 3\int e^x\,dx = \ln|x| + 3e^x + C$$

$$\int_1^2 \frac{4}{x^2}\,dx = 4\int_1^2 x^{-2}\,dx = 4\left[-x^{-1}\right]\Big|_1^2 = 4\left[\frac{-1}{2} - \frac{-1}{1}\right]$$

$$= 4\left[\frac{-1}{2} + 1\right] = 2$$

$$\int_0^\pi \cos y + \sin y\,dy = \int_0^\pi \cos y\,dy + \int_0^\pi \sin y\,dy = \sin y\Big|_0^\pi + (-\cos y)\Big|_0^\pi$$

$$= \sin(\pi) - \sin(0) - \cos(\pi) + \cos(0)$$

$$= 0 - 0 - (-1) + 1 = 2$$

$$\int_{-a}^a x^5\,dx = \frac{1}{6}x^6\Big|_{-a}^a = \frac{1}{6}a^6 - \frac{1}{6}(-a)^6 = 0$$

OR

Odd function over even limits $= 0$

Problem 2 (0 points)

Find the value of

$$\int_2^5 f(x)\,dx$$

using the following information:

$$\int_5^0 3f(x)\,dx = 9.$$ —— this tells me $-3\int_0^5 f(x)\,dx = 9$ so $\int_0^5 f(x)\,dx = -3$

$$\int_0^2 f(x)\,dx = 5$$ —— use this and rules about limits of integration

then $\underbrace{\int_0^5 f(x)\,dx}_{=-3} = \underbrace{\int_0^2 f(x)\,dx}_{=5} + \int_2^5 f(x)\,dx$

so $-3 = 5 + \int_2^5 f(x)\,dx$

$\int_2^5 f(x)\,dx = -3 - 5 = -8$

Problem 3 (0 points)

Explain in words what the integral represents, give units.

$$\int_0^2 f(x)\,dx$$

where $f(x)$ is the rate at which you drink coffee the morning before an exam, measured in gallons per minute and x is measured in minutes, with $x = 0$ corresponding to 6am when you wake up.

This is the amount of coffee that you drink totale from 6am to 8am. Totale coffe in the first two hours measured in gallons

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

where $f(x)$ is the amount of money, measured in dollars, in your bank account on day x.

This is the average amount of money in dollars that I have between days a and b.

Problem 4 (0 points) Find the **general antiderivative** of the following functions:

(a) $f(x) = x^2$ → $F(x) = \frac{x^3}{3} + C$ — need the constant!
(b) $g(x) = 2\sin x$ → $F(x) = -2\cos(x) + C$
(c) $h(x) = e^x + 2x + 1$ → $F(x) = e^x + x^2 + x + C$

I can check my answers by taking the derivative of F to see that I get f in each case.

Problem 5 (0 points)

Given $f(x) = x^3 + 3x^2 - 9x + 6$, do the following:

(a) Find all local maxima and minima.

$f'(\bar{x}) = 3\bar{x}^2 + 6\bar{x} - 9 = 0$ critical points.

$x = 1, -3$ are critical points.

$3(x^2 + 2x - 3) = 3(x+3)(x-1) = 0$

Test $\longrightarrow$ $f''(x) = 6x + 6$

so $f''(1) = 12 > 0$ $f''(-3) = -12 < 0$

$x = 1$ is a MIN

$x = -3$ is a MAX

local

(b) Find all inflection points

$f''(x) = 6x + 6 = 0$ so $x = -1$ is an inflection point.

(c) Now consider the domain $0 \leq x \leq 2$. Find the global maximum and minimum.

$f(0) = 6$ $f(2) = 8 + 12 - 18 + 6 = 8$

$f(1) = 1 + 3 - 9 + 6 = 1$

Global Min $f(1) = 1$ Global Max $f(2) = 8$

(d) For the domain $0 \leq x \leq 2$, what are the best possible bounds?

$1 \leq y \leq 8$ or $1 \leq f(x) \leq 8$

(e) Estimate the area under the curve using the average of the LEFT and RIGHT hand sums, with $0 \leq x \leq 4$ and $n = 4$. (SHOW YOUR WORK)

$\int_0^4 x^3 + 3x^2 - 9x + 6\,dx$ if $n = 4$ then $\Delta x = \frac{4-0}{4} = 1$

need $f(0) = 6$ $f(3) = 33$

$f(1) = 1$ $f(4) = 82$

$f(2) = 8$

LHS $= [6 + 1 + 8 + 33]1 = 48$

RHS $= [1 + 8 + 33 + 82]1 = 124$

Average $= \frac{124 + 48}{2} = 86$

(f) Find the exact value of $\int_0^4 f(x)\,dx$ using the antiderivative and the fundamental theorem of calculus. (SHOW YOUR WORK)

$$\int_0^4 x^3 + 3x^2 - 9x + 6\,dx = \frac{x^4}{4} + \frac{3x^3}{3} - \frac{9x^2}{2} + 6x \Big|_0^4$$

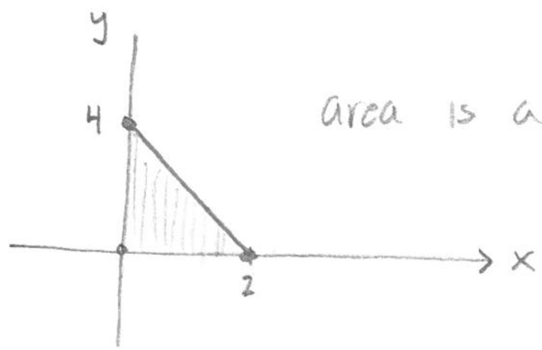
$$= \frac{4^4}{4} + 4^3 - \frac{9(4)^2}{2} + 6(4) = 80$$

Problem 6 (0 points)

Find the area under the line $y = -2x + 4$ and above the x axis for the domain $0 \leq x \leq 2$ by setting up and evaluating a definite integral. Then use basic geometry to verify that your answer is correct, by thinking of the area in terms of rectangles and/or triangles. (Draw a picture)

$$\int_0^2 -2x + 4\,dx = -x^2 + 4x\Big|_0^2 = -(2)^2 + 4(2) - \emptyset$$
$$= -4 + 8 = 4$$

Draw $y = -2x + 4$ between 0 and 2



area is a triangle

Area $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2)(4) = 4$ ✓

Problem 7 (0 points)

for now... assume constant unless otherwise noted

A rocket traveling at $87m/s$ is accelerated to $132m/s$ over a time of 15 seconds. What is it's displacement during this time?

total acceleration $= \frac{132-87}{15} = 3\ m/s^2$

velocity $\frac{dv}{dt} = a = 3$ so $v(t) = 3t + C$ but $v(0) = \boxed{C = 87}$ $V = 3t + 87$

displacement $\frac{ds}{dt} = V = 3t + 87$ $\int_0^{15} 3t + 87\, dt = \frac{3t^2}{2} + 87t \Big|_0^{15} = 1642.5\ m.$

A pilot stops a plane in 484 meters using an acceleration of $-8m/s^2$. How fast was the plane moving before braking began? How long did it take the plane to come to a complete stop?

$\frac{dv}{dt} = -8$ and displacement = 484 meters

$V(0) = V_0$

we will solve for this!

$V(t) = -8t + C$ $V(0) = C = V_0$ <u>so</u> $V(t) = -8t + V_0$

At stop time t_s ... $V(t_s) = 0$

so $\boxed{V_0 = 8t_s}$

then displacement:

$$\int_0^{t_s} -8t + V_0\, dt = -4t^2 + V_0 t \Big|_0^{t_s} = \boxed{-4t_s^2 + V_0 t_s = 484}$$

plug in V_0

$-4t_s^2 + 8t_s^2 = 484$

or $t_s^2 = \frac{484}{4}$ or $t_s = \pm 11$

choose positive

The plane stops in 11 seconds.

meaning $V_0 = 8t_s = 88$

It was going 88 m/s.