MATH 121
Practice Problems for Midterm 2

Name: ______________________________

Score	
1	/0
2	/0
3	/0
4	/0
5	/0
6	/0
7	/0
Total	/0

These practice problems are not intended to be an exhaustive list of problems for the exam.

Any problem type that we have done on the homework or in class is fair game for the exam. You should go watch the homework videos to see some great solutions!

The exam will consist of just 3-4 randomly assigned problems.

If you understand all of the concepts you see here then you should be prepared for the exam. If you memorize the problems here, you will not be prepared for the exam, because the problems will be different.

On the exam you should show as much work as possible to get as much partial credit as possible. I am very generous with partial credit when it is clear that students are making an effort to explain themselves.

Things you should know for this exam:

All of the basic algebra rules for functions.
All of the derivative shortcut rules!
What the derivative means. This includes saying in words what $f(x)$, $f'(x)$ and $f''(x)$ means given some information about $f(x)$.
How to combine the rules to take more complicated derivatives, including using the inverse.
Derivatives of implicit functions
How to identify the derivatives on a graph.
Slope and Curvature as they are related to the first and second derivatives.
Finding the equation of a tangent line.
Using the tangent to approximate a function near a point. Eg. Given $f'(a)$ and $f(a)$ what is an approximate value of $f(a + .5)$
Using derivatives to evaluate limits of more complicated functions.

Problem 1 (0 points)

Calculate the derivative of the following functions using the shortcuts learned in class:

$$f(x) = x^2 + 3^x - e^x + 4x^2 - 1$$

$$f(y) = \frac{y^2 - y + 1}{y}$$

$$f(x) = (x^3 + x - 9)e^x$$

$$f(x) = (x^3 + x - 9)^{20}$$

$$g(\theta) = \sin(\sqrt{\theta})$$

$$s(t) = e^{\tan(t)}$$

$$f(x) = \arctan(x^3 + e^x)$$

Problem 2 (0 points)

For each of the problems below do the following:

a. Find all points on the curve with the given y value.

b. Graph the function on Desmos.com and check that these points are on the curve. Do you expect slopes to be positive or negative at these points?

c. Find the derivative $\frac{dy}{dx}$.

d. Find the equation of the tangent line for each of the points you found in part 1.

e. Check your tangent lines by adding them to the graph you made in part 2.

f. Are there any locations where the slope is zero or undefined?

1. $x^2 + xy - y^3 = xy$ and $y = 1$
2. $\sin(xy) = 2x + 5$ and $y = 0$
3. $x^2y - 2y + 5 = 0$ and $y = 5$

Problem 3 (0 points)

Using the functions plotted on the next page, say if the following are positive, negative, zero, or undefined. You MUST explain how you came to your conclusion.

(a) $f(3)$

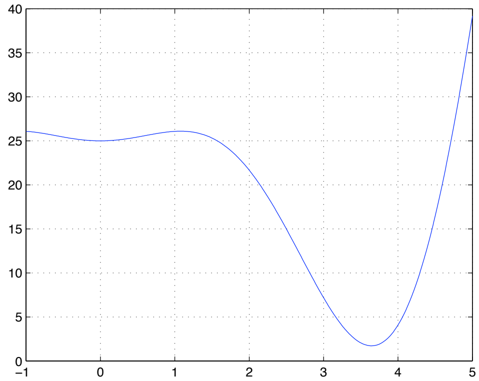
(b) $f'(3)$

(c) $f''(3)$

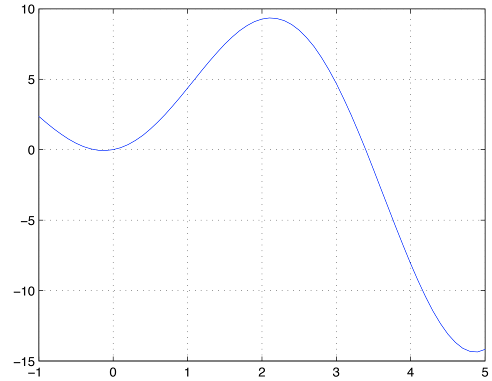
(d) $\frac{d}{dx}\left(f(3)g(3)\right)$

(e) $\frac{d}{dx}\left(c \cdot g(3)\right)$ where c is a positive constant.

(f) $\frac{d}{dx}\left(g(f(3))\right)$



(a) A graph of $g(x)$



(b) A graph of $f(x)$

Problem 4 (0 points)

Consider the equation $f(x) = x^3 + 3x^2 + 4$.

(a) Find the equation of the tangent line at $x = 0$.

(b) Where is $f(x)$ concave up?

(c) Where is $f(x)$ is increasing?
HINT: There should be two conditions $x > a$ and $x < b$ where a and b are numbers that you find. Plug in numbers if you're not sure.

Problem 5 (0 points)

Consider the function $f(x) = \sqrt{1+x}$, your goal is to find a linear approximation for $f(0.1)$. Use the following steps:

1. What "easy" point is near the point $x = .1$ where we would like to make our approximation? In other words, what x value near $x = 0.1$ would be WAY easier to plug into our function?

2. Find the tangent line approximation for $f(x)$ near the point $x = a$, where a is the "easy" point that you just found.

3. Now plug the point $x = 0.1$ into the tangent line approximation you just found. This gives you your answer "an approximate value for $f(0.1)$"

4. Now USING YOUR CALCULATOR FOR THE FIRST TIME.... find the "real" value for $f(0.1) = \sqrt{1.1}$. How close was your answer?

Problem 6 (0 points)

Evaluate the following limits. If you use L'Hopitals rule indicate how the hypotheses are met.

$$\lim_{x \to 0} \frac{\ln(1+7x)}{2x}$$

$$\lim_{x \to \pi} \frac{\sin(x)}{x - \pi}$$

$$\lim_{x \to 1} \frac{x^2 - 1}{1 + \ln(x)}$$

$$\lim_{x \to \infty} \frac{x^2}{\ln(x)}$$

Problem 7 (0 points)

Extra Challenge Problem - on some exams I include a challenge problem that is more difficult but not worth a ton of points. This problem is the difference between an A and and A+ on the exam. Here are some examples:

1. Prove the power rule for non integer values of n using inverses, the chain rule, and the power rule.

2. Find the derivative of

$$f(x) = \arccos(x)$$

using inverses. Simplify your answer using trigonometric rules.

3. Show that you can get the quotient rule using the product rule.