Problem 5 (0 points)

1. For $f(x) = x^3 - 6x^2 + 9x + 2$

$f'(x) = 3x^2 - 12x + 9$
$3(x^2 - 4x + 3) = 0 \qquad 3(x-3)(x-1) = 0$
$x = 3 \quad x = 1$

(a) find all local extrema by using the **first derivative test**. Then check your answer using the **second derivative test**.

$x=2$	$x=3$	$x=4$
$-$	0	$+$
$\searrow$	$-$	$\nearrow$ MIN

$x=0$	$x=1$	$x=2$
$+$	0	$-$
$\nearrow$	$-$	$\searrow$ MAX

$\boxed{\begin{array}{l} x=3 \text{ is a MIN} \\ x=1 \text{ is a MAX} \end{array}}$

$f''(x) = 6x - 12$

$f''(3) = 18 - 12 = 6$
concave up
MIN!

$f''(1) = 6 - 12 = -6$
concave dow
MAX!

(b) Are there any inflection points? You must show your work to get credit.

$f''(x) = 6x - 12 = 0 \qquad 6x = 12$

set second derivative equal to zero $\rightarrow$ $x = 2$ is an inflection point

2. Find the best possible bounds for the function xe^{-2x}, $0 \le x < \infty$.

Find Global Max and Min.

Critical Points: $f' = e^{-2x} - 2xe^{-2x} = (1-2x)e^{-2x} = 0$
$x = 1/2$
$f(\tfrac{1}{2}) = \tfrac{1}{2}e^{-1}$

Check endpoints:
$f(0) = 0e^0 = 0$

$\lim_{x\to\infty} xe^{-2x} = \lim_{x\to\infty} \dfrac{x}{e^{+2x}} = \lim_{x\to\infty} \dfrac{1}{+2e^{+2x}} = 0$

USE l'Hopitals rule to evaluate the limit.

LOWER BOUND $f(x) = 0$ UPPER BOUND $f(x) = \frac{1}{2}e^{-1}$

3. Find the Global Max and Min for the following functions:

$f(x) = x^4 - 8x^2$

$f'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 0 \qquad x = \pm 2$, $x = 0$
$f(0) = 0 \quad f(2) = 16 - 8\cdot 4$
$= 16 - 36 = -16 \quad f(-2) = -16$

$\lim_{x\to\infty} f(x) = \infty \qquad \lim_{x\to-\infty} f(x) = \infty$

$\boxed{\begin{array}{l} \text{GLOBAL MIN } x = -16 \\ \text{GLOBAL MAX NONE} \end{array}}$

$f(x) = \dfrac{x+1}{x^2+3}$

$f'(x) = \dfrac{1(x^2+3) - (x+1)(2x)}{(x^2+3)^2}$

$\dfrac{x^2 + 3 - 2x^2 - 2x}{x^4 + 6x^2 + 9} = \dfrac{-x^2 + 3 - 2x}{x^4 + 6x^2 + 9}$

$x^2 + 2x - 3 = 0$
$(x+3)(x-1) = 0$
$x = -3 \quad x = 1$

$f(1) = \dfrac{2}{4} \qquad f(-3) = \dfrac{-2}{12} = \dfrac{-1}{6}$

$\lim_{x\to\infty} f(x) = 0$
$\lim_{x\to-\infty} f(x) = 0$

$\boxed{\begin{array}{l} \text{GLOBAL MAX } x = \frac{2}{4} = \frac{1}{2} \\ \text{GLOBAL MIN} = -1/6 \end{array}}$

Problem 6 (0 points)

1. A ball is thrown up into the air so that it's position in meters at time t seconds is given by $f(t) = 80t - 16t^2$. Answer the following questions about the ball:

(a) What do $f(1) = 64$ and $f'(1) = 48$ mean?

after one second the ball is 64 meters in the air / and at that moment its velocity is $48\ m/s^2$

(b) What is velocity of the ball at time $t = 2$?

$$f'(t) = 80 - 36t \quad f'(2) = 80 - 36(2) = 80 - 72 = \boxed{8\ m/s}$$

(c) At what time does the ball reach it's maximum height?

$$80 - 36t = 0 \qquad t = \frac{80}{36} = \frac{40}{18} = \boxed{\frac{20}{9} \text{ seconds}}$$

2. A spherical balloon is inflated so that its radius is increasing at a constant rate of 1 cm/s. At what rate is its volume increasing when the raduis is $5cm$. NOTE: Volume of a sphere $\frac{4}{3}\pi r^3$.

$$\frac{dr}{dt} = 1\ cm/s \qquad r = 5cm$$

$$V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

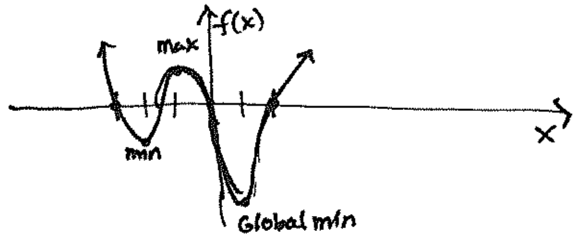
$$= 4\pi (5)^2(1) =$$

$$= 100\pi$$

Problem 7 (0 points)

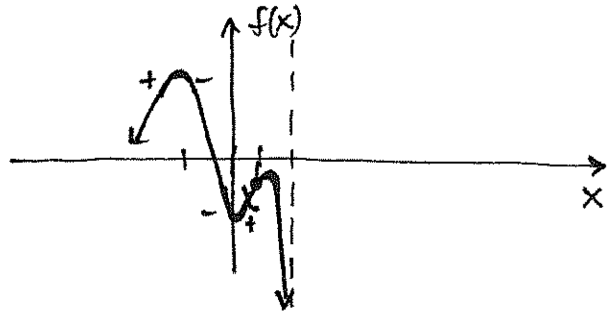
1. Using the following information, draw a sketch of the functions:

(a) $f(x)$ has a local minimum at $x=-2$ and global minimum at $x=1$ and local maximum at $x=-1$. It crosses the x-axis at $x=-3$ and $x=2$ and has inflection points at $x=1.5$ and $x=0$.



Inflection – try to show going from concave up to concave down

(b) $f'(x)$ changes from positive to negative at $x=-1$, changes from negative to positive at $x=0$ and is undefined at $x=1$. It also has an inflection point at $x=0.5$.



Just a Sketch
your graph might look different

2. Find the values of a and b so that the function $y = axe^{-bx}$ has a local maximum at the point $(2, 10)$.

$$y' = ae^{-bx} - abxe^{-bx} = (1-bx)ae^{-bx} = 0 \quad x = 1/b$$

max @ $x=2$ so $\boxed{b = 1/2}$

	$x=1$	$x=2$	$x=3$
$(1-\frac{1}{2}x)=$	+	0	−

MAX

$$y = axe^{-1/2x}$$

$$10 = a2e^{-1}$$

$$\frac{10}{2e^{-1}} = a \quad \boxed{a = 5e}$$

$$y = 5exe^{-1/2x} = 5xe^{-1/2x+1}$$

3. For a positive constant, a, find the critical points of the function $f(x) = xe^{-ax}$. For what value of a does the function have a critical point at $x=2$? Is this critical point a max or a min?

$$f'(x) = e^{-ax} - axe^{-ax} = (1-ax)e^{-ax} = 0 \quad \boxed{x = 1/a}$$

$$x = 2 \Rightarrow \boxed{a = 1/2}$$

This would be a MAX

	$x=1$	$x=2$	$x=3$
$(1-\frac{1}{2}x)$	+	0	−

Problem 8 (0 points)

1. The total cost C(q) of producing q goods is given by $C(q) = .5q^2 + 2q + 10$. The company who produces this product can sell each one for 42 dollars. If Profit = Revanue - Cost, then what quantity of goods should they produce to maximize profit?

Cost $= C(q) = .6q^2 + 2q + 10$

Revenue $= R(q) = 42q$

$P = -.6q^2 - 2q - 10 + 42q$

$= 40q - .5q^2 - 10$

$P' = 40 - 2 \cdot \frac{1}{2} q$ $\quad q = \frac{40}{1} = 40$

2. A closed cylinder with radius r has a surface area of $8cm^2$. What is the maximum volume for this cylinder? NOTE Volume $= \pi r^2 h$ and Surface Area $= 2\pi r h + 2\pi r^2$.

$2\pi r h + 2\pi r^2 = 8 \qquad h = \frac{8 - 2\pi r^2}{2\pi r}$

$V = \pi r^2 h = \cancel{\pi} r^{\cancel{2}} \cdot \frac{8 - 2\pi r^2}{2\cancel{\pi}\cancel{r}} = \frac{r}{2}(8 - 2\pi r^2)$

$= 4r - \pi r^3$

$\frac{dV}{dr} = 4 - 3\pi r^2 = 0 \qquad r^2 = \frac{4}{3\pi} \qquad r = \pm \frac{2}{\sqrt{3\pi}}$

$r = \frac{2}{\sqrt{3\pi}}$

Max Volume:

plug back in...

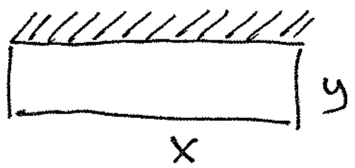
$V = 4r - \pi r^3$

$= \frac{8}{\sqrt{3\pi}} - \frac{8\pi}{3\pi\sqrt{3\pi}} = \frac{24\pi - 8\pi}{3\pi\sqrt{3\pi}}$

$= \frac{16}{3\sqrt{3\pi}}$

Problem 9 (0 points)

1. If you have 100 feet of fencing and you want to enclose a rectangular area against a long, straight, wall, what is the largest area that you can enclose?



Amt of fencing given

$2y + x = 100$

solve for x

$x = 100 - 2y$

WANT MAX AREA

sub in

$\text{Area} = xy = (100-2y)y = 100y - 2y^2$

Take derivative and find critical points

$\frac{dA}{dy} = 100 - 4y \qquad y = \frac{100}{4} = 25$

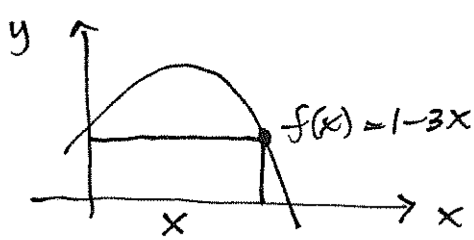
$x = 100 - 2(25) = 50$

$A'' = -4 < 0$ MAX

Answer final question

$A = 25 \times 50 = 1550$

2. A ~~square~~ rectangle has one side along the x-axis, one side on the y-axis, and one vertex on the function $f(x) = 1 - 3x^2$. What is the maximum area for this ~~square~~ rectangle?



$\text{Area} = x \cdot y = x(1-3x^2)$

$= x - 3x^3$

$\frac{dA}{dx} = 1 - 9x^2 \qquad x^2 = \frac{1}{9}$

$x = \pm \frac{1}{3}$

$A'' = -18x$

$x = \frac{1}{3}$ then $A'' < 0$

$\cap$ MAX

Area = [redacted]

$= \frac{1}{3} - 3\left(\frac{1}{3}\right)^3$

$= \frac{2}{9}$

Problem 10 (0 points)

CHALLENGE PROBLEM:

You run a small furniture business. You sign a deal with a customer to deliver up to 400 chairs, the exact number to be determined by the customer later. The price will be \$90 per chair up to 300 chairs and will be reduced by \$0.25 per chair (on the whole order) for every additional chair over 300 ordered. What are the largest and smallest revenues your company can make under this deal?

$0 \leq c \leq 400$

90 per upto 300
price reduction after 300

Assume more than 300 chairs.

$c = 300 \quad R = 90 \times 300$

$c = 301 \quad R = (90 - .25) \times 301 \implies R = \left(90 - \frac{1}{4}(c - 300)\right) \times c$

$c = 302 \quad R = (90 - .25(2)) \times 302$

$$R = 90c - \frac{1}{4}c^2 + 75c$$

$$= 165c - \frac{1}{4}c^2$$

$R' = 165 - \frac{1}{2}c = 0 \qquad c = 2(165)$

$R'' = -\frac{1}{2} < 0 \quad \wedge \qquad = 330$ chairs.

THIS IS MAX

$$R = \left(90 - \frac{1}{4}30\right) \times 330 = \$27225$$

TEST $R(0) = 0 \qquad R(400) = (90 - 25) \times 400 = 65 \times 400 = \$25600$

$R(300) = \$27000$

MIN $R = \$0$ no chairs.

MAX $R = \$27225$