MATH 121
Practice Problems for Midterm 1

Name: ______________________________

Score	
1	/5
2	/5
3	/5
4	/5
5	/5
6	/5
7	/5
Total	/35

These practice problems are not intended to be an exhaustive list of problems for the exam.

Any problem type that we have done on the homework or in class is fair game for the exam. You should go watch the homework videos to see some great solutions!

The exam will consist of just 3-4 randomly assigned problems.

If you understand all of the concepts you see here then you should be prepared for the exam. If you memorize the problems here, you will not be prepared for the exam, because the problems will be different.

On the exam you should show as much work as possible to get as much partial credit as possible. I am very generous with partial credit when it is clear that students are making an effort to explain themselves.

Things you should know for this exam:
** All of the basic algebra rules for functions.
** How to take a limit of a function.
** What it means for a function to be continuous and/or differentiable.
** Definition of average and instantaneous velocity.
** The central difference approximation for estimating the derivative.
** How to find a derivative algebraically using the limit and how to find a derivative numerically using data.
** What the derivative means. This includes saying in words what $f(x)$, $f'(x)$ and $f''(x)$ means given some information about $f(x)$.
** How to identify the derivatives on a graph.
** Slope and Curvature as they are related to the first and second derivatives.
** Finding the equation of a tangent line.
** Using the derivative to approximate a function near a point. Eg. Given $f'(a)$ and $f(a)$ what is an approximate value of $f(a+.5)$

Problem 1 (5 points)

(a) Draw two functions: one that is continuous and one that is not continuous. Use the formal definition of continuity to describe why one is continuous everywhere and why the other is not.
(b) Draw a function that is continuous but NOT differentiable. Explain in words why the function is continuous but not differentiable.
(c) Draw a function that has the following qualities:

$$\lim_{x\to 0^+} f(x) = 0 \qquad lim_{x\to 0^-} f(x) = 1 \qquad f'(2) = -1$$

(d) Sketch the function $y = x^2 - 4$ on your sketch lines on the graph that represent the following items:

- $f'(0)$
- $f'(1)$
- The average rate of change between $f(0)$ and $f(1)$

For each of the lines you drew explain WHY it represents the given item and give the exact value for the calculation.

Problem 2 (5 points)

Assume that $f(t) = (t+1)^2$ gives the position of an object at time t. Assume units of meters and seconds.

(a) Find the average velocity between $t = 1$ and $t = 2$.

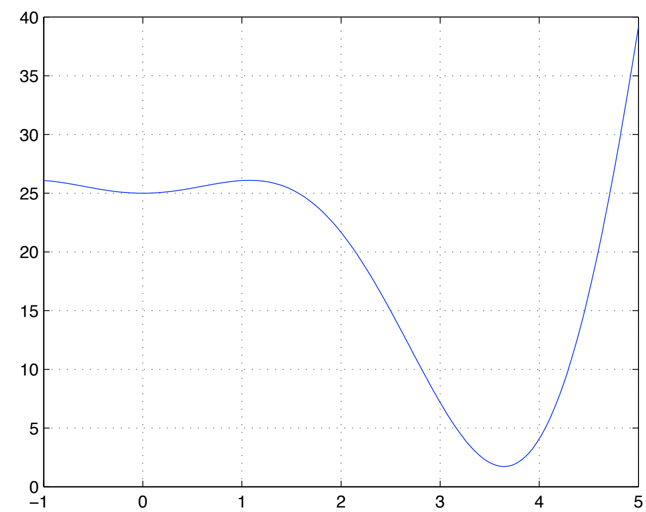
(b) Find the instantaneous velocity at $t = 1$.

(c) Find the time when the velocity is exactly zero.

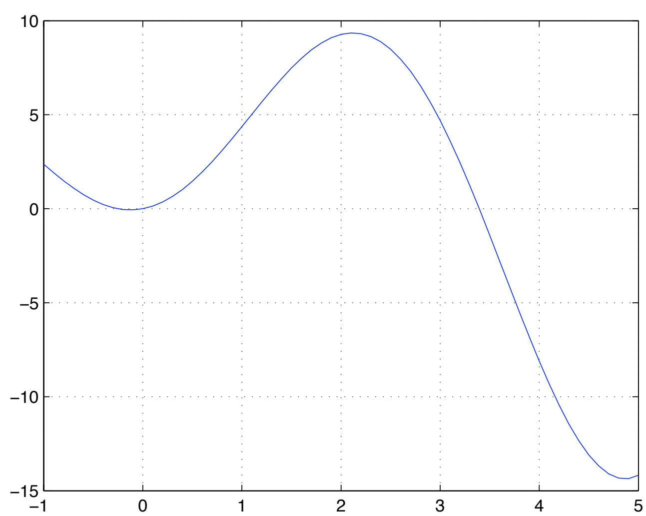
(d) For what times is the velocity increasing or decreasing?

Problem 3 (5 points)

(a) On the graph below: Put a dot everywhere **the derivative** is zero, a + everywhere the derivative is positive, and a − everywhere the derivative is negative. Explain in words how you decided to make these marks.



(b) On the graph below: Put a dot everywhere **the second derivative** is zero, a + everywhere the derivative is positive, and a − everywhere the derivative is negative. Explain in words how you decided to make these marks.



Problem 4 (5 points)

The function $G(c)$ represents the average grade on this test as a function of the number of the number of cookies that you bake for your professor (to eat while she's grading). Here c is the number of cookies and G is the average grade measured in percent. For example, $G(1)$ is the average grade on the exam if you baked one cookie for your professor. Explain the meaning of the following:

(a) $G(11) = 90$
(b) $G'(c)$, include units.
(c) $G'(11) = 1$, include units.
(d) Estimate the value of $G(12)$ using the information above.

Problem 5 (5 points)
Consider the function $f(x) = x^2 + x$

(a) Find the value of $f'(3)$ **numerically or computationally** by setting up a table of values to evaluate the limit.

(b) Find estimate the value of $f'(3)$ using the **central difference approximation**.

(c) Find the derivative function $f'(x)$ **algebraically**, using the limit definition of the derivative. What is the true value of $f'(3)$? How close were your numerical and estimated values?

(d) Find the second derivative function $f''(x)$, using the limit definition of the second derivative.

Problem 6 (5 points)
Given the function $f(x)$ in the picture below, find the x-value where the following things are true and say in words why you chose that value.

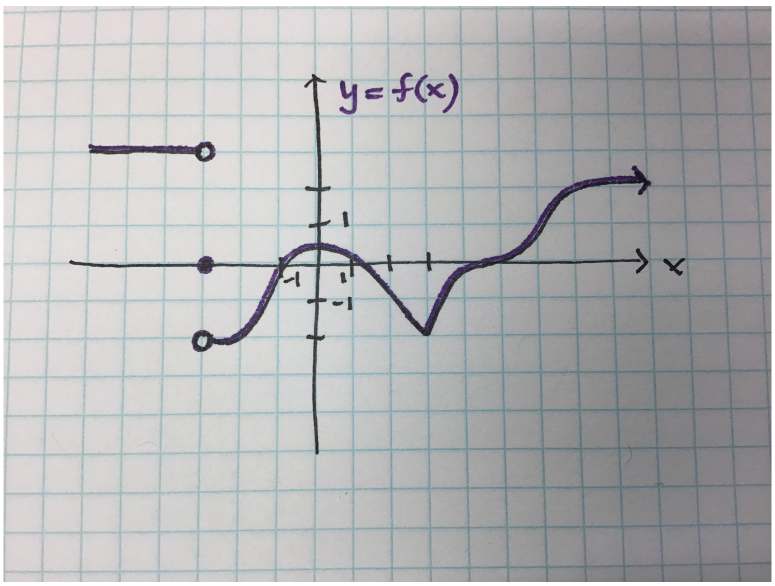
(a) A location where the limit of $f(x)$ does not exist.

(b) A location where $f'(x) = 0$

(c) A location where the function is continuous but not differentiable

(d) A location where $f''(x)$ is negative

(e) A two points a and b where the average rate of change of $f(x)$ is $AV_{[a,b]} = 2$



Problem 7 (5 points)
Every morning I feed my cats Bella and Bongo breakfast. I bring the food over, ask them to sit (which they do!) and then put the two identical bowls down. The following data was taken as I watched my cats eat their morning food. I measured the height of the food in the bowl as a function of time.

Bella's Food

Time t (seconds)	0	10	20	30	40	50	60
Height HBE(t) (centimeters)	4	3	2.25	1.75	1.5	1	.9

Bongo's Food

Time t (seconds)	0	10	20	30	40	50	60
Height HBO(t) (centimeters)	4	2	1.5	1.5	1.5	1.5	1.5

Please answer the following questions:

(a) Are the derivatives $HBE'(t)$ and $HBO'(t)$ ever positive or negative or zero? Explain what each would mean.

(b) What are the units of the derivative? What does the derivative mean in terms of the real world cats.

(c) Estimate each derivative using the method of your choice

$HBE'(10)$

$HBO'(10)$

$HBE'(50)$

$HBO'(50)$

(d) What do the functions and derivatives mean in terms of each cat's eating style? Who eats faster? Who eats more?

(e) Estimate using the derivative at $t = 50$

$HBE(70)$

and

$HBO(70)$

explain why you think your estimate is a good one.