Calculus 1 - Riemann Sums

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PART 1: For each function listed below please do the following:

1. Find an estimate for the Left and Right Hand sum using $n = 5$ subdivisions. Write down $LHS =$ and $RHS =$. (Do this by hand.)
2. Draw a graph of the actual function and plot the Left and Right hand estimates that you found above. Be sure to clearly label your graph.
3. How far apart are your estimates? Write down difference=
4. Now use the website: Riemann Sum Applet - Click Here to answer the rest of the questions.
   - Enter the function in the integrand
   - Move the blue dots on the graph to match the limits of integration
   - Move the slider to the right number of subintervals
   - Check the box next to "Relative" and use the relative approximation when the slider is all the way left and all the way right.
5. Find an estimate for the Left and Right Hand sum using $n = 10, 15, 20$ subdivisions. Write down $LHS =$ and $RHS =$.
6. How far apart are your estimates with the larger n-values? Write down the difference.
7. Compare the values you get to the actual value of the integral given.
8. As you increase the number of subintervals is your estimate getting better or worse? How could we get an exact answer.

A.

$$f(x) = \sin(x) \;\; 0 \leq x \leq 5$$

Check your answer using the real values:

$$\int_0^5 \sin(x)\, dx = 0.71634$$

B.

$$f(x) = -x^2 + 4 \;\; 0 \leq x \leq 2$$

Check your answer using the real values:

$$\int_0^2 (-x^2 + 4)\, dx = 5.3333333$$

C.

$$f(x) = e^{-x} \;\; 0 \leq x \leq 10$$

Check your answer using the real values:

$$\int_0^{10} e^{-x}\, dx = 0.9995$$

PART 2: Knowing that the integral represents the area under the curve, answer the following questions.

A. Consider the equation for the upper half circle of radius $r = 1$ given by $y = \sqrt{(1 - x^2)}$. What should the actual value of the following integral be:

$$\int_{-1}^{1} \sqrt{(1 - x^2)}\ \, dx =$$

Compute the LHS and RHS for $-1 \leq x \leq 1$ choosing just one reasonable n-value. Does this match with what you know the answer should be?

B. Consider the integral:

$$\int_0^{2\pi} \sin(x)\, dx =$$

What should the value of this integral be? HINT: Graph it! Compute the LHS and RHS for $0 \leq x \leq 2\pi$ choosing just one reasonable n-value. Does this match with what you know the answer should be?