Calculus II - Integration Practice - Solutions

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1. $\int (2t+7)^{72}\ dt$

   Do a substitution. Let $u = 2t+7$ then $du = 2dt$ and subbing in we find $\displaystyle\int \frac{1}{2}u^{72}\ du = \frac{1}{2}\frac{1}{73}(2t+7)^{73}$

2. $\int y\left(\ln(y)\right)\ dy$

   Integration by parts. Let $u = \ln(y)$ and $dv = y$ then $du = \frac{1}{y}dy$ and $v = \frac{1}{2}y^2$. Subbing into the integration by parts formula we get $\frac{1}{2}y^2\ln(y) - \frac{1}{2}\int y\ dy = \frac{1}{2}y^2\ln(y) - \frac{1}{4}y^2 + c$

3. $\displaystyle\int \frac{x+1}{x^2+6x}\ dx$

   Use partial fractions. Factor the denominator into $x(6+x)$ then write
   $$\frac{x+1}{x(6+x)} = \frac{A}{x} + \frac{B}{6+x}$$
   Multiply through by the denominator from the LHS $x+1 = A(6+x)+Bx$ and gather terms $x+1 = (A+B)x+6A$ then balancing the sides wer see that $(A+B) = 1 \rightarrow A = 1-B$ and $6A = 1 \rightarrow A = \frac{1}{6}\quad B = \frac{5}{6}$ so we can rewrite our integral as
   $$\frac{1}{6}\int \frac{1}{x}\ dx + \frac{5}{6}\int \frac{1}{6+x}$$
   giving a solution $\frac{1}{6}\ln|x| + \frac{5}{6}\ln|6+x| + c$

4. . $\displaystyle\int \frac{1}{x^2+4x+4}\ dx$

   Here we can use algebra to simplify the denominator. $x^2+4x+4 = (x+2)^2$ so our integral is $\int \frac{1}{(x+2)^2}\ dx = \int (x+2)^{-2}\ dx = -\frac{1}{x+2} + c$

5. $\displaystyle\int \frac{x}{(1+x^2)^2}\ dx$

   Substitution. Let $u = 1+x^2$ then $du = 2x\ \ dx$ and we see that the x in the numerator will cancel to give us $\int \frac{1}{2u^2}\ du = \frac{-1}{u} + c = \frac{-1}{x+2} + c$

6. $\int se^{s^2}\ ds$

   Substitution. Let $u = s^2$ then $du = 2s\ ds$ and subbing in this gives $\int \frac{1}{2}e^u\ du = \frac{1}{2}e^{s^2} + C$

7. $\displaystyle\int \frac{r^3+r^2+1}{r}\ dr$

   Algebra helps us simplify: $\int r^2 + r + \frac{1}{r}\ dr = \frac{1}{3}r^3 + \frac{1}{2}r^2 + \ln|r| + c$

8. $\int t\sin(t^2)\ dt$

   Do a substitution. Let $u = t^2$ then $du = 2tdt$ and substituting we find $\int \frac{1}{2}\sin(u)\ du = \frac{-1}{2}\cos t^2 + c$

9. $\int x \sin(x)\ dx$
Integration by parts:
let $u = x$ and $du = \sin x dx$. Subbing into the formula we get $-x \cos x + \int \cos x\ dx = -x\cos x + \sin x + c$.

10. $\int (t+2)\sqrt{2+3t}\ dt$
Do a substitution. Let $u = 2+3t$ then $du = 3dt$. After some simplification this leads to
$\frac{1}{9}\int (u^{(\frac{3}{2})} + 4u^{(\frac{1}{2})} =$
$\frac{2}{45}(2+3t)^{(\frac{5}{2})} + \frac{8}{27}(2+3t)^{(\frac{3}{2})} + c$

11. $\int \frac{x^3+7x^2+10x+1}{x^2+7x+10}\ dx$
Long division gives us:
$\int x\ dx + \int \frac{1}{(x^2+7x+10)}\ dx$ then we can evaluate the first integral and do partial fractions to find
$\frac{1}{2}x^2 - \frac{1}{3}\int \frac{1}{x+5}\ dx + \frac{1}{3}\int \frac{1}{x+2}\ dx =$
$\frac{1}{2}x^2 + \frac{1}{3}\ln\left|\frac{x+2}{x+5}\right| + c$

12. $\int e^y \cos(y)\ dy$
Do integration by parts twice and then solve for the original integral to get $\frac{1}{2}e^y\left[\sin(y) + \cos(y)\right] + c$

13. $\int \frac{p^2+2}{p^2+p}\ dp$
Long division gives $\int dp + \int \frac{2-p}{p^2+p}\ dp$. Then do partial fractions to get
$\int dp + \int \frac{2}{p}\ dp - \int \frac{3}{p+1}\ dp =$
$p + 2\ln|p| - 3\ln|p+1| + c$

14. $\int \frac{x^3 \sin(x)}{x \sin(x)}\ dx$
This one simplifies nicely to $\int x^2\ dx = \frac{1}{3}x^3 + c$

15. $\int \cos^4(t)\ dt$
Use the trick for even powers of sine or cosine. Here we use the identity $\cos^2(t) = \frac{1}{2}(1+\cos(2t))$ which gives
$\int \cos^2(t)\cos^2(t)\ dt = \int \frac{1}{2}(1+cos(2t))\frac{1}{2}(1+cos(2t))\ dt =$
$\frac{1}{4}\int 1 + 2\cos(2t) + cos^2(2t)\ dt.$
The first two integrals we can just do, but we need to use the identity again for the third term in the integrand to get
$\frac{1}{4}\left[t + \sin(2t) + \frac{1}{2}\int(1+\cos(4t)\right]\ dt =$
$\frac{1}{4}\left[t + \sin(2t) + \frac{1}{2}(t + \frac{1}{4}\sin(4t))\right] + c$

16. $\int s^2\left(1+2s^3\right)^2\ ds$
Do a substitution. Let $u = 1+2s^3$ then $du = 6s^2 ds$ and subbing in gives
$\int \frac{1}{6}u^2\ du = \frac{1}{18}\left(1+2s^3\right)^3 + c$

17. $\int \sin^3(x)\ dx$
Use the trick for odd powers of sine and consine. Break off the odd piece to get
$\int \sin(x)\sin^2(x)\ dx$ then use the identity
$\sin^2(x) + \cos^2(x) = 1$ to find

$\int \sin(x)(1 - cos^2(x))\ dx$ which allows for a substitution. Let $u = \cos(x)$ then $du = \sin(x)dx$ and subbing in gives $\int (1 - u^2)\ du = \cos(x) - \frac{1}{3}\cos^3(t) + c$

18. $\int \sin(x)\sec(x)\ dx$
Use the identity $sec(x) = \frac{1}{\cos(x)}$ and then substitute $u = \cos(x)$ to get
$-\int \frac{1}{u}\ du = -\ln|\cos(x)| + c$.

19. $\int \sec^2(y)\ dy$
This is a really weird substitution, but a good one to know... Let $u = \tan x$ then $du = \sec^2(x)\ dx$ so the whole integrand becomes
$\int du = u + c = \tan(x) + c$ Alternatively you could just remember that the derivative of tangent is secant squared: $\frac{d}{dx}\tan(x) = \frac{1}{cos^2(x)} = \sec^2(x)$ (proved by the derivative quotient rule) and then you just know the answer.

20. $\int x^2 e^x\ dx$
Do integration by parts twice. Let $u = x^2$ and $dv = e^x dx$. After subbing into the formula we get: $x^2e^x - \int 2xe^x\ dx$. Then $u = 2x$ and $dv = e^x$ to get
$x^2e^x - 2xe^x + 2e^x + c$