Calculus II Class 2 (Class 1 Review of Calc)
Lecture 1

- ~~Hand in Review of Calc~~
- ~~Homework 1 posted on BB Due Wednesday 1/19~~
- ~~Review materials from Calc I online~~

~~OFFICE HOURS AHN 229~~
~~MWF 1-2:30~~
~~TTh 11-1~~

5.6 Integration by substitution.

- Finding derivatives is pretty straightforward - product rule quotient rule, chain rule.
- Make sure that you are awesome at finding derivatives ... this is importand in integration.

We can solve simple integrals just by knowing the antiderivative.

$$\int x^2\,dx = \frac{x^3}{3} + C$$ possible because we know that $\frac{d}{dx}\left[\frac{x^3}{3}\right] = x^2$

What about more complicated integrals?

$$\int 3x^2 \cos(x^3)\,dx$$

Guess and check ... $\cos(x^3)$? $\frac{d}{dx}\left[\cos(x^3)\right] = \sin(x^3)\,3x^2$ ✗

$\sin(x^3)$? $\frac{d}{dx}\left[\sin(x^3)\right] = \cos(x^3)\cdot 3x^2$ ✓

This is a valid but time consuming way to solve integrals!

The Method of Substitution — systematic way to solve integrals of the form

$$\int f(g(x))\cdot g'(x)\,dx$$

WE ARE UNDOING THE CHAIN RULE!

THE METHOD OF SUBSTITUTION:

<u>Ex</u> $\int 3x^2\cos(x^3)\,dx$

1. Look at the integral and choose a substitution.
   "What is the most difficult/troblesome part of the integrand?"

   let $w = x^3$

   - choose the inside function for the substitution.
   - I pick this because I want something more simple inside the cosine
   - Also I see that w' is also part of the integral

2. Solve for the differential

   $dw = 3x^2\,dx$ so $dx = \dfrac{dw}{3x^2}$

   solve for dx.

3. Sub everything into the integrand.

$$\int 3x^2\cos(x^3)\,dx = \int 3x^2\cos(w)\,\frac{dw}{3x^2} = \int \cos(w)\,dw$$

   - I usually expect some cancelation or simplification
   - final integral SHOULD NOT contain x.

4. Solve the easier Integral

$$\int \cos(w)\,dw = \sin(w) + c$$

5. Substitute back in to get final solution in terms of the correct variable

$$\sin(w) + c = \sin(x^3) + c$$

<u>so</u> $$\int 3x^2\cos(x^3)\,dx = \sin(x^3) + c$$

<u>Ex</u> $\int te^{(t^2+1)}\,dt$ | see a function inside a function ... try substitution.

1. Let $w = t^2+1$ — this is the inside function, the thing really causing problems.

2. Find expression for dt:

$$dw = 2t\,dt \quad \text{so} \quad dt = \frac{dw}{2t}$$

3. Sub in to Integrand:

$$\int te^{(t^2+1)}\,dt = \int t e^{w}\,\frac{dw}{2t} = \int \frac{1}{2} e^{w}\,dw$$

4. Solve $\frac{1}{2}\int e^{w}\,dw = \frac{1}{2}e^{w} + C$

5. Sub back in for w: $\boxed{\int te^{(t^2+1)}\,dt = \frac{1}{2}e^{t^2+1} + C}$

YOU TRY...

$$\int x^3\sqrt{x^4+5}\,dx = \frac{1}{4}\left[\frac{2}{3}u^{3/2} + C\right] = \frac{1}{6}(x^4+5)^{3/2} + C$$

$$\int e^{\cos\theta}\sin\theta\,d\theta = -e^{\cos\theta} + C$$

$$\int \frac{e^t}{1+e^t}\,dt = \ln|1+e^t| + C$$

Problems that could arrise ...

Ex $\int x^2\sqrt{x^4+5}\,dx$ this is similar to the problem you tried...

normally... let $w = x^4+5$
then $dw = 4x^3dx$ and $dx = \frac{dw}{4x^3}$

plug in:

$$\int x^2\sqrt{x^4+5}\,dx = \int x^2\sqrt{w}\,\frac{dw}{4x^3} = \int \frac{1}{4x}\sqrt{w}\,dw$$

This final integrall MUST NOT contain x ... either substitution wont work or we need to find some substitution for x...

IF THINGS DONT CANCEL OUT...

1. Check your algebra or derivatives - can you sub more in?
2. Check your substitution - could you make another guess?
3. Try another method.

Ex $\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\,d\theta$

often w/ trigonometric functions rewriting helps!

let $u = \cos\theta$
then $du = -\sin\theta\,d\theta$ and $d\theta = \frac{-du}{\sin\theta}$

plug in... $\int \frac{\sin\theta}{\cos\theta}\,d\theta = \int \frac{\sin\theta}{u}\left(\frac{-du}{\sin\theta}\right) = -\int \frac{1}{u}\,du$

$$= -\ln|u| + C$$

sub back to original vars... $\int \tan\theta\,d\theta = -\ln|\cos\theta| + C$

this would have been hard to guess!!

How do we deal w/ definite integrals?

Only difference: you also need to transform the limits of integration.

Ex $\displaystyle\int_0^2 x e^{x^2}\,dx$

Start the substitution like normal
let $u = x^2$
then $du = 2x\,dx$ and $dx = \dfrac{du}{2x}$

NOW CONVERT THE LIMITS! Limits: $0 \le x \le 2$

when $x = 0$ $u = (0)^2 = 0$ $\Rightarrow$ $0 \le u \le 4$
$x = 2$ $u = (2)^2 = 4$
Keep limits in order...

Sub everything in and use new limits.

$$\int_0^2 x e^{x^2}\,dx = \int_0^4 x e^{u}\,\frac{du}{2x} = \frac{1}{2}\int_0^4 e^{u}\,du$$

now evaluate: $= \frac{1}{2}e^{u}\Big|_0^4 = \frac{1}{2}e^4 - \frac{1}{2}e^0$

$= \frac{1}{2}\left[e^4 - 1\right]$

no need to sub back in... our answer is a number!

YOU TRY...

$\displaystyle\int_1^3 \frac{dx}{5-x} = -\ln|2| + \ln|4|$

section 1 Stop
Finish This @ home!

$\displaystyle\int_0^{\pi/4} \frac{\tan^3\theta}{\cos^2\theta}\,d\theta$ Hint: two possible choices
$w = \tan\theta$ $w = \cos\theta$
look @ dw to see which works better at canceling things out!

$= 1/4$

A MORE COMPLEX PROBLEM...

EX $\int \sqrt{1+\sqrt{x}}\ dx$

the problem is the mess under the square root. we will try to sub this out!

let $w = 1+\sqrt{x}$

then $dw = \dfrac{dx}{2\sqrt{x}}$ $\quad dx = 2\sqrt{x}\, dw$

sub back in

$$\int \sqrt{1+\sqrt{x}}\, dx = \int \sqrt{w} \cdot 2\sqrt{x}\, dw$$

are there any problems here?

but wait... $\sqrt{x} = w-1$

$$= \int 2\sqrt{w}\,(w-1)\, dw = \int 2w^{3/2} - 2w^{1/2}\, dw$$

much easier to deal with!

$$= 2\left[\tfrac{2}{5} w^{5/2}\right] - 2\left[\tfrac{2}{3} w^{3/2}\right] + C$$

$$= \tfrac{4}{5}\left[1+\sqrt{x}\right]^{5/2} - \tfrac{4}{3}\left[1+\sqrt{x}\right]^{3/2} + C$$

YOU TRY

$$\int (x+7)\sqrt[3]{3-2x}\ dx.$$