- HW 1 due Wed. → questions ?
- HW 2 due Friday assigned today
- Office Hrs. MWF 1–2:30
  TTh 11–1
- Tutoring Hours Sun – Thurs 7:30 – 9:30
  AHN 216.

LAST TIME... Integration by Substitution.

Integrals of the form $\int f'(g(x))g'(x)\,dx$

- function inside a function.

Ex. $\int (x+7)\sqrt[3]{3-2x}\,dx$ (harder case)

1. Choose a substitution

let $w = 3-2x$

then $\frac{dw}{dx} = -2$ $\quad dx = -\frac{dw}{2}$

2. find dx.

3. Sub in...

$$\int (x+7)\sqrt[3]{w}\left(-\frac{dw}{2}\right) = -\frac{1}{2}\int (x+7)\sqrt[3]{w}\,dw$$

↑ hmm is this a problem?

- if $w = 3-2x$

then $x = \frac{1}{2}(3-w) = \frac{3}{2} - \frac{w}{2}$

$$= -\frac{1}{2}\int \left(\frac{3}{2} - \frac{w}{2} + 7\right)\sqrt[3]{w}\,dw$$

$$= -\frac{1}{2}\int \left(\frac{17}{2} - \frac{w}{2}\right)\sqrt[3]{w}\,dw = -\frac{1}{4}\int 17w^{1/3} - w^{4/3}\,dw$$

$$= -\frac{1}{4}\left[17\frac{3}{4}w^{4/3} - \frac{3}{7}w^{7/3} + C\right] = -\frac{1}{4}\left[\frac{51}{4}w^{4/3} - \frac{3}{7}w^{7/3}\right] + C.$$

4. Get back to original variables...

$$\int (x+7)\sqrt[3]{3-2x}\,dx = -\frac{1}{4}\left[\frac{51}{4}(3-2x)^{4/3} - \frac{3}{7}(3-2x)^{7/3}\right] + C.$$

Today a new awesome technique!

7.1 Integration By Parts: Our Goal ... undo the product rule.

lets look at
the product rule ... $(uv)' = u'v + v'u$

or $\frac{d}{dx}(uv) = \frac{du}{dx}v + \frac{dv}{dx}u$

lets integrate this

$$\int \frac{d}{dx}(uv)\,dx = \int \frac{du}{dx} v\,dx + \int \frac{dv}{du} u\,dx$$

$$uv = \int v\,du + \int u\,dv$$

OR $\boxed{\int u\,dv = uv - \int v\,du}$ This is the Formula for Integration by parts!

1. Choose a u and a dv from the original integral
2. From those find v and du
3. Plug into integration by parts formula
4. Now solve $\int v\,du$ instead.

GOAL... $\int v\,du$ should be easier than $\int u\,dv$!!

Ex $\int xe^x\,dx$ • wouldn't it be nice if we didn't have that x...

let $u = x$ and $dv = e^x dx$ 1 choose u and dv

then $du = dx$ and $v = e^x$ 2. find du and v
derivative of u integrate dv

3. Sub In... $\int xe^x = xe^x - \int e^x\,dx$ much easier to integrate!

$= xe^x - e^x + C$ 4. Solve the easier integral!

<u>Ex</u> $\int \theta \cos\theta \, d\theta$ • I see two functions multiplied ... if I take the derivative of one it gets simpler!

let $u = \theta$ and $dv = \cos\theta \, d\theta$ 1. Choose u and dv

then $du = d\theta$ and $v = \sin\theta$ 2. Find du and v

$$\int \theta\cos\theta \, d\theta = \theta\sin\theta - \int \sin\theta \, d\theta = \theta\sin\theta + \cos\theta + C$$

3. Sub into Formula 4. solve easier Integral.

What if we made the wrong choice? ... no big deal! Lets see what happens.

$$\int \theta \cos\theta \, d\theta$$

let $u = \cos\theta$ and $dv = \theta \, d\theta$

then $du = -\sin\theta \, d\theta$ and $v = \frac{1}{2}\theta^2$

$$= \tfrac{1}{2}\theta^2\cos\theta + \frac{1}{2}\int \theta^2 \sin\theta \, d\theta$$

hmm this is NOT easier !!

1. For u choose the thing that reduces upon taking a derivative ex. x^n, $\ln(x)$, ...
2. For dv choose the thing that remains simple upon Integration ex. $\sin x$, $\cos x$, e^x

<u>You try:</u> $\int x \sin x \, dx$.

$\int \ln(x) \, dx$ hint... let $u = \ln(x)$

$\int x\sin x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + c$

let $u = x$ $\quad dv = \sin x\,dx$

$du = dx$ $\quad v = -\cos x$

$\int \ln(x)\,dx = x\ln(x) - \int dx = x\ln(x) - x + c$

let $u = \ln(x)$ $\quad dv = dx$

$du = \frac{1}{x}dx$ $\quad v = x$

How does this change for Definite Integrals?

$\int_2^3 \ln(x)\,dx$

basically the same ... just remember to apply limits of integration.

let $u = \ln(x)$ $\quad dv = dx$

$du = \frac{1}{x}dx$ $\quad v = x$

$$= x\ln(x)\Big|_2^3 - \int_2^3 dx = x\ln(x)\Big|_2^3 - x\Big|_2^3$$

$$= 3\ln(3) - 2\ln(2) - 3 + 2$$

$$= 3\ln(3) - 2\ln(2) - 1$$

our solution is a #!

SOME TRICKS ...

Ex $\int x^3\ln(x)\,dx$

It is tempting to choose $u = x^3$ and $dv = \ln(x)dx$

BUT $\int \ln(x)\,dx$ does not make things easier.

let $u = \ln(x)$ $\quad dv = x^3dx$

$du = \frac{1}{x}dx$ $\quad v = \frac{x^4}{4}$

<u>NOTE</u> u is not always the first function!

$$= \frac{x^4}{4}\ln(x) - \int \frac{x^3}{4}dx = \frac{x^4}{4}\ln(x) - \frac{x^4}{16} + c$$

Ex $\int x^2 \cos 2x \, dx$

let $u = x^2$ $\quad dv = \cos 2x \, dx$

then $du = 2x \, dx$ $\quad V = \frac{\sin 2x}{2}$

$$= \frac{x^2 \sin 2x}{2} - \int x \sin 2x \, dx$$

well it is simpler but we're not done yet !!

we need to integrate by parts again! OK to do it more than once.

let $u = x$ $\quad dv = \sin 2x \, dx$

then $du = dx$ and $V = -\frac{\cos 2x}{2}$

$$= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \int \frac{\cos 2x}{2} \, dx$$

$$= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4} + C$$

How many times would we need to do integration by parts on:

$\int x^{16} \sin x \, dx$? SIXTEEN TIMES !

YOU TRY $\int x^2 e^x \, dx = x^2 e^x - 2\int x e^x \, dx = x^2 e^x - 2x e^x + 2\int e^x \, dx$

$$= \boxed{x^2 e^x - 2x e^x + 2e^x + C}$$