MATH 122
Practice Exam 3 **Joanna Bieri**

Name: ______________________________

Instructions:

1. Print your name on this page in the space provided.
2. **You must CIRCLE your FINAL answer for full credit.**
3. Show all work, write down the formulas used and explain in words what you are doing, partial credit will be given for written work only. Answers with no work will NOT be given full credit. **Neatness counts.**
4. Use of notes or books is NOT ALLOWED.
5. **You may have one sheet of paper for notes and formulas and a calculator**
6. Good luck!

Score

1	/0
2	/0
3	/0
4	/0
5	/0
6	/0
Total	/0

Important Ideas on this exam:

1. Calculating Limits and Convergence of Sequences
2. Convergence Test for Series (Integral, Comparison, Limit Comparison, Ratio, Alternating Series Test, Divergence Test, etc)
3. Important Series: Geometric, P-Series, Taylor Series Formulas.
4. How to prove the a Series or Power Series converges/diverges.
5. Given a Function, find the Taylor Series (or Power Series)
6. Find the Radius and Interval of Convergence
7. Use Taylor Polynomials to approximate functions and analyze the error.

Problem 1 (0 points)

Find the Taylor Series centered at $c = 0$ for the function $f(x) = \cos(x^2)$.

Now integrate this series to find $\int_0^1 \cos(x^2)\, dx$. Your answer should be an infinite series.

Now approximate this integral using the first four terms in the series you just found.

More Practice: Redo the problem for $f(x) = \frac{\cos(x)}{x}$ and $f(x) = \sin(x^3)$.

Problem 2 (0 points)

Consider the sequence $a_n = \frac{n}{n^2+1}$.

(a) Does the **sequence** a_n converge or diverge? What does the limit as $n \to \infty$ of this sequence tell you about $\sum_{n=1}^{\infty} a_n$? What does it tell you about $\sum_{n=1}^{\infty}(-1)^n a_n$?

(b) Make a conjecture: Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge?

(c) Prove, using a comparison or limit comparison test, your conjecture from part (b).

(d) What does the ratio test tell you about $\sum_{n=1}^{\infty} a_n$?

More Practice: Redo the problem for $a_n = \frac{1}{3^n+1}$, $a_n = \frac{1}{n^4+3n^3+7}$, and $a_n = \frac{5n+1}{3n^2-1}$

Problem 3 (0 points)

For the power series given below, find the Radius of Convergence and the Interval of Convergence.

$$\sum_{n=0}^{\infty} nx^n$$

More Practice: $\sum_{n=0}^{\infty}(2^n + n^2)x^n$ and $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n2^n}$

Problem 4 (0 points)

Consider the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$$

(a) What does the Leibniz Test say about the infinite series? Explain your answer.

(b) Does the series converge absolutely? Show mathematical work to back up your answer.

<u>Problem 5</u> (0 points)
Use the integral test to prove whether the following series converge or diverge.

$$\sum_{n=0}^{\infty} \frac{1}{(n+2)^2}$$

$$\sum_{n=0}^{\infty} \frac{1}{e^n}$$

Problem 6 (0 points)

(a) Show that the Taylor Series for $f(x) = e^x$ is given by $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$.

(b) Write down the Taylor Polynomial centered at $x = 0$ that approximates $e^{-0.2}$ to six decimal places. You will need to use the Lagrange Error Bound and a calculator.

(c) Use the Taylor Series in part (a) to solve the following integral:

$$\int_0^1 e^{-x^2}\, dx$$