MATH 122
PRACTICE Exam 1

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Name: Solutions - Practice Exam.

Instructions:

1. Print your name on this page in the space provided.
2. **You must put your answer in the box provided.**
3. Show all work, **write down the formulas used**, partial credit will be given for written work only. **Answers with no work will not be given full credit. Neatness counts.**
4. Use of notes or books is not allowed.
5. Calculators are not allowed.
6. Good luck!

Score	
1	/20
2	/30
3	/20
4	/20
5	/30
6	/30
7	/30
8	/20
9	/0
Total	/200

Problem 1 (20 points)

Use the partial fractions above to evaluate:

$$\int \frac{1}{x^3 - 9x^2}\, dx$$

Consider the Integrand:

$$\frac{1}{x^3 - 9x^2} = \frac{1}{x^2(x-9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-9}$$

Factor

Separate Fractions.

Then $1 = Ax(x-9) + B(x-9) + Cx^2$

$1 = Ax^2 - 9Ax + Bx - 9B + Cx^2$

$1 = (A+C)x^2 + (B-9)x - 9B$

So $A + C = 0$ $\quad B - 9A = 0$ $\quad -9B = 1$

$C = 1/81$ $\quad -1/9 - 9A = 0$ $\quad B = -1/9$

$-9A = 1/9$

$A = -1/81$

$$\int \frac{1}{x^3 - 9x^2}\, dx = \int \frac{-1/81}{x}\, dx - \int \frac{1/9}{x^2}\, dx + \int \frac{1/81}{x-9}\, dx$$

$$= -1/81 \ln|x| + 1/9\, x^{-1} + 1/81 \ln|x-9| + C$$

Problem 2 (30 points)

Evaluate the following integrals using integration by parts.

$$\int \underbrace{y}_{u}\,\underbrace{e^{3y}}_{dv}\,dy$$

u		dv
y	$+$	e^{3y}
1	$-$	$\frac{1}{3}e^{3y}$
0	$+$	$\frac{1}{9}e^{3y}$

Table Method.

$$\int ye^{3y}dy = \frac{y}{3}e^{3y} - \frac{1}{9}e^{3y} + c$$

$$\int x\underbrace{ln(x)}_{\text{choose } ln(x) = u}\,dx$$

let $u = ln(x)$ $\quad dv = x$

$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$$= \frac{1}{2}x^2 ln|x| - \frac{1}{2}\int x\,dx = \frac{1}{2}x^2 ln|x| - \frac{1}{4}x^2 + c$$

Problem 3 (20 points)

Evaluate the following improper integral.

$$\int_0^\infty xe^{-x^2}\,dx$$ improper because ∞ appears in limits.

Consider

$$\int_0^b xe^{-x^2}\,dx$$ Consider the Integral you Can Solve!

let $u = -x^2$ Substitution.
$du = -2x\,dx$
$dx = \frac{-du}{2x}$

when $x=0$ $u=0$ $\rightarrow$ IMPORTANT !!
$x=b$ $u=-b^2$

$$= -\int_0^{-b^2} \tfrac{1}{2}e^u\,du = -\tfrac{1}{2}e^u\Big|_0^{-b^2} = -\tfrac{1}{2}e^{-b^2} + \tfrac{1}{2}e^0$$ Evaluate the Integral.

$$\lim_{b\to\infty} \tfrac{1}{2}\left[1 - e^{-b^2}\right] = \tfrac{1}{2}$$ Take The Limit.

so $$\int_0^\infty xe^{-x^2}\,dx = \tfrac{1}{2}$$

converges to $1/2$.

Problem 4 (20 points)

Complete the square and solve the resulting integral.

$$\int \frac{1}{y^2+2y+5}\,dx$$

[HINT: You will need to use a trigonometric substitution to evaluate the integral, make sure to draw your triangle]

$y^2+2y+5=(x+b)^2+c$

$x^2+2bx+b^2+c$

$2=2b \quad b=1 \qquad b^2+c=5$

$1+c=5$

$c=4$

$y^2+2y+5=(x+1)^2+4$

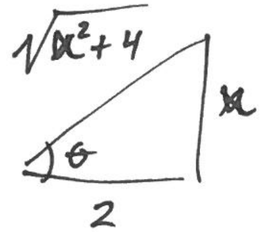
Transform the integrand.

$\int \frac{1}{(x+1)^2+4}\,dx = \int \frac{1}{x^2+4}\,dx$

let $u=x+1$

$du=dx$

TRIG SUB.



$x=2\tan\theta \qquad \cos\theta=\frac{2}{\sqrt{x^2+4}}$

$dx=\frac{2\,d\theta}{\cos^2\theta} \qquad \frac{1}{4}\cos^2\theta=\frac{1}{x^2+4}$

$=\int \frac{1}{4}\cos^2\theta\cdot\frac{2\,d\theta}{\cos^2\theta}=\frac{1}{2}\int d\theta=\frac{1}{2}\theta+c$

$=\frac{1}{2}\arctan\left(\frac{u}{2}\right)+c=\frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+c$

Problem 5 (30 points)

Evaluate the following integrals using substitution.

$$\int y^7(y^8+8)^{32}\,dy \longrightarrow = \frac{1}{8}\int u^{32}\,du = \frac{1}{8}\cdot\frac{1}{33}u^{33}+c$$

let $u = y^8+8$

$du = 8y^7\,dy$

$dy = \frac{du}{8y^7}$

$$= \frac{1}{264}(y^8+8)^{33}+c$$

$$\int_0^1 \frac{\cos(x)}{1-\sin(x)}\,dx$$

let $u=\sin(x)$ OR let $u = 1-\sin(x)$ ✓easier.

$du = -\cos(x)\,dx$

$dx = \frac{-du}{\cos(x)}$

when $x=0$ $u=1$

$x=1$ $u=1-\sin(1)$

$$-\int_1^{1-\sin(1)} \frac{1}{u}\,du = -\ln|u|\Big|_1^{1-\sin(1)}$$

$$= -\ln|1-\sin(1)| + \ln|1| = -\ln|1-\sin(1)|$$

Problem 6 (30 points)

Evaluate the following indefinite integrals using the method of your choice. You must state what method you are using!

$\int e^y \cos(y)\, dy$ Integration by parts twice to get a copy.

let $u = \cos(y)$ $dv = e^y\, dy$

$du = -\sin(y)\, dy$ $v = e^y$

$= e^y \cos(y) + \int e^y \sin y\, dy$

$u = \sin y$ $dv = e^y dy$

$du = \cos y\, dy$ $v = e^y$

$= e^y \cos(y) + e^y \sin(y) - \int e^y \cos(y)\, dy$ so

solve for this.

$2\int e^y \cos(y)\, dy = e^y \cos(y) + e^y \sin y$

$\int x^2 e^{x^3}\, dx$

AND $\int e^y \cos(y)\, dy = \frac{1}{2} e^y [\cos(y) + \sin(y)] + c$

Substitution.

let $u = x^3$ $du = 3x^2 dx$

$dx = \frac{du}{3x^2}$

$= \int \frac{1}{3} e^u\, du = \frac{1}{3} e^u + c = \frac{1}{3} e^{x^3} + c$

Problem 7 (30 points)

Evaluate the following definite integrals using the method of your choice. You must state what method you are using!

$$\int_0^3 \frac{1}{(x-2)^2}dx = \int_0^2 \frac{1}{(x-2)^2}dx + \int_2^3 \frac{1}{(x-2)^2}dx$$

could do a u-sub
but this is one we know... problem when $x=2$
IMPROPER!

$$\int_0^b \frac{1}{(x-2)^2}dx = \frac{-1}{x-2}\Big|_0^b = \frac{-1}{b-2} + \frac{1}{-2} = \frac{-1}{b-2} - \frac{1}{2} \qquad \lim_{b\to 2^-} \frac{-1}{b-2} - \frac{1}{2} = \infty$$

$$\int_a^3 \frac{1}{(x-2)^2}dx = \frac{-1}{x-2}\Big|_a^3 = -1 + \frac{1}{a-2} = \frac{1}{a-2} - 1 \qquad \lim_{a\to 2^+} \frac{1}{a-2} - 1 = \infty$$

$\int_0^3 \frac{1}{(x-2)^2}dx$ diverges.

$$\int_0^1 x\sin(x^2)\,dx$$

let $u = x^2$
$du = 2x\,dx$
when $x=0$ $u=0$
$x=1$ $u=1$

$$\longrightarrow \int_0^1 \frac{1}{2}\sin u\,du = -\frac{1}{2}\cos(u)\Big|_0^1$$

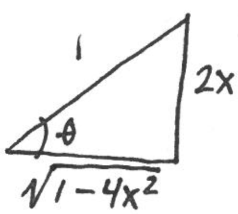
$$= -\frac{1}{2}\cos(1) + \cos(0)$$

$$= 1 - \frac{1}{2}\cos(1).$$

Problem 8 (20 points)

Evaluate the following integral using trigonometric substitution.

$$\int \frac{1}{\sqrt{1-4x^2}}\,dx$$



$\sin\theta = \frac{2x}{1}$

$$\boxed{\begin{aligned} x &= \tfrac{1}{2}\sin\theta \\ dx &= \tfrac{1}{2}\cos\theta\, d\theta \end{aligned}}$$

sub in

$$\int \frac{1}{\cancel{\cos\theta}} \cdot \frac{1}{2}\cancel{\cos\theta}\, d\theta$$

$\cos\theta = \sqrt{1-4x^2}$

$$\boxed{\text{so } \frac{1}{\sqrt{1-4x^2}} = \frac{1}{\cos\theta}}$$

$$\int \frac{1}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\int d\theta = \frac{1}{2}\theta + C = \frac{1}{2}\arcsin(2x) + C$$

Problem 9 (0 points)
EXTRA CREDIT 5 POINTS

Starting from the product rule for differentiation, derive the formula for integration by parts. You must show all your work to get credit, no partial credit.

Product Rule:

$$\frac{d}{dx}(uv) = \frac{du}{dx}\cdot v + u\frac{dv}{dx}$$

integrate... $\int \frac{d}{dx}uv\,dx = \int \frac{du}{dx}\cdot v\,dx + \int u\frac{dv}{dx}dx$

simplify...

$$uv = \int v\,du + \int u\,dv$$

solve for this!

$$\int u\,dv = uv - \int v\,du$$

Integration by Parts Formula!