MATH 122
Practice Exam 2 **Joanna Bieri**

Name: ______________________________

Score	
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
11	/10
Total	/110

Instructions:
This is a set of practice problems for Exam 2.

If you are able to do all these problems without looking at books, notes, OR SOLUTIONS then you should be prepared for the exam. You should know that these problems are representative of the type of problems on the exam, not just a copy of the exam problems with some constants changed. You must understand the underlying methods to do well on the exam.

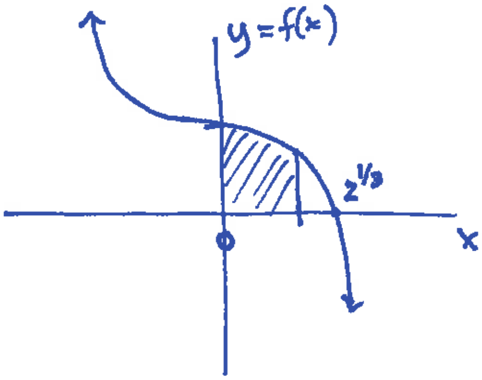
There are more problems here than there will be on the exam.

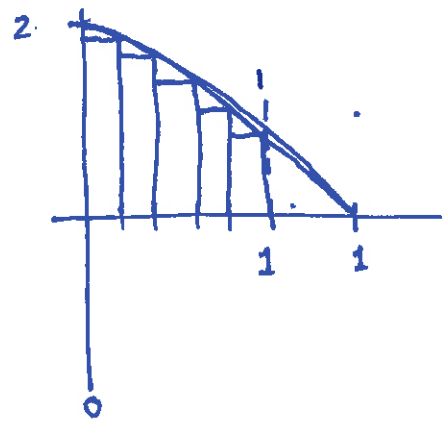
Also make sure you can do all the homework problems and example problems from the class notes.
Important Ideas:

1. You should be able to use any of the General Integration Methods from Chapter 7 - Substitution, Integration by Parts, Trigonometric Substitution, Algebraic Simplification, Powers of sin(x) and cos(x), Methods of Approximation.
2. Finding Area and Volume using Integration.
3. Volumes of Revolution.
4. Applications to Physics: Mass, Work, and Pressure.
5. Integration in Polar Coordinates.
6. Numerical Integration - Approximations and Error

[redacted]

Sketch the function $f(x) = -x^3 + 2$, then answer the following questions:





a. How would you use ~~the midpoint rule~~ RIGHT(5) to evaluate $\int_0^1 f(x)\,dx$? Sketch ~~MID~~ Right(5) on your graph of $f(x)$.

Divide the region into 5 boxes taking the height of the box from the right hand side.

b. Would ~~MID~~ Right(5) be an underestimate or overestimate of the value of the integral?

It would be an underestimate because the boxes are all beneath the curve.

c. How would you find ~~SIMP~~ MID(5)? (Just say in words what you would have to do.)

Instead take the height of the box from a point in the middle of the range.

To bound the error:

RIGHT $\rightarrow$ $|E_R| \leq \frac{K_1(b-a)^2}{2n}$ $\quad K_1 = \max_{\text{on}[a,b]} |f'(x)|$ $\quad |f'(x)| = |-3x^2|$ this is biggest when $x=1$

$= \frac{3(1-0)^2}{2 \cdot 5} = \frac{3}{10}$ $\quad$ So $K_1 = 3$

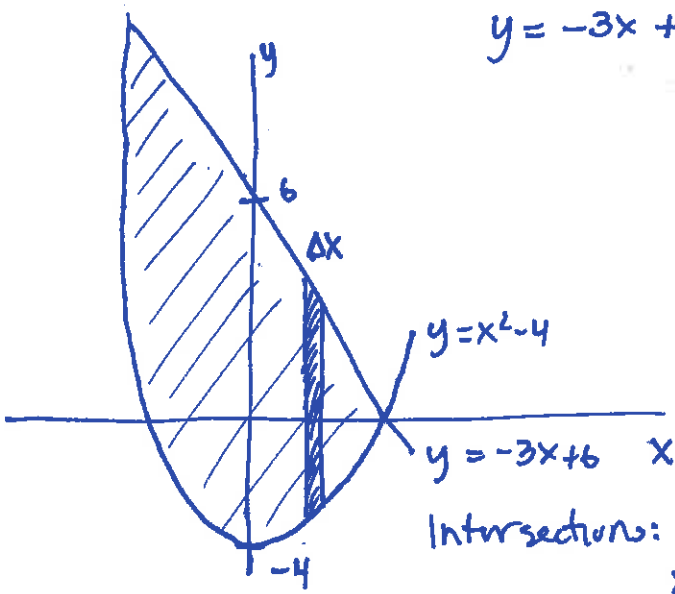
MID $\rightarrow$ $|E_M| \leq \frac{K_2(b-a)^3}{24n^2}$ $\quad K_2 = \max_{\text{on}[a,b]} |f''(x)|$ $\quad |f''(x)| = |-6x|$ biggest when $x=1$

$= \frac{6(1-0)^2}{24(5)^2} = \frac{6}{24 \cdot 25}$ $\quad$ So $K_2 = 6$

[redacted])

Draw the area bounded by the curves $3x + y = 6$ and $y = x^2 - 4$, then use integration to find the value for the area bounded by the two curves.

$y = -3x + 6$



Slice:

$A_i = -3x+6-x^2+4 \; \Delta x$

add all the pieces.

Intersections: $x^2-4 = -3x+6$

$x^2+3x-10=0 \quad (x+5)(x-2)=0$

$$\text{Area}_T = \lim_{n\to\infty} \sum_{i=1}^{n} -3x_i+10-x_i^2 \, \Delta x = \int_{-5}^{2} 10-3x-x^2 \, dx$$

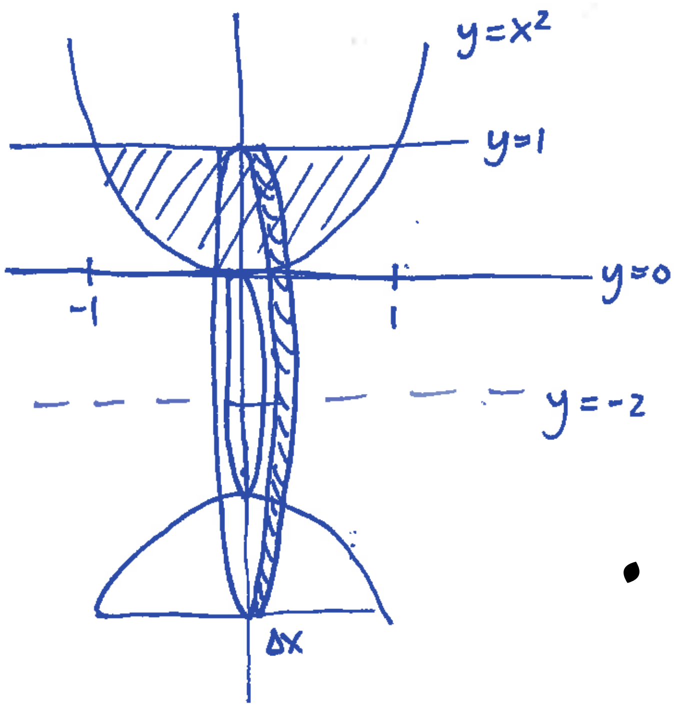
$$= 10x - \frac{3}{2}x^2 - \frac{x^3}{3} \Big|_{-5}^{2} = 10(2)-10(-5)-\frac{3}{2}(2)^2+\frac{3}{2}(-5)^2 - \frac{(2)^3}{3} - \frac{(-5)^3}{3}$$

$$= 20+50 + -6 + \frac{3(25)}{2} - \frac{8}{3} + \frac{125}{3}$$

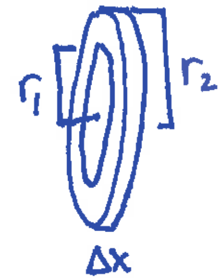
[redacted]

[redacted]s)

Find the volume of the solid generated by rotating the region bounded by $y = x^2$, $y = 0$, $y = 1$ about the line $y = -2$.



Slice:



$$V_i = \pi(r_2^2 - r_1^2)\Delta x$$

$$r_2 = 1-(-2) = 3$$

$$r_1 = x^2-(-2) = x^2+2$$

$$V_i = \pi\left((x^2+2)^2 - 9\right)\Delta x$$

$$V_{TOTAL} = \pi \int_{-1}^{1} \left[(x^2+2)^2 + 9\right] dx = \pi \int_{-1}^{1} -x^4 \mp 4x^2 \mp 4 + 9 \, dx$$

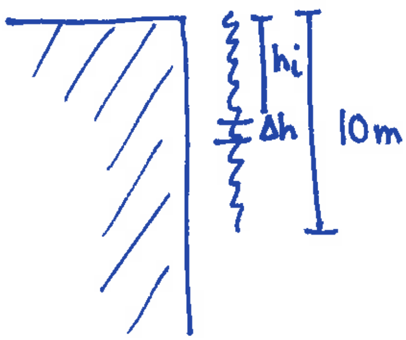
$$= -\pi \int_{-1}^{1} x^4 + 4x^2 - 5 \, dx = -\pi \left[\frac{x^5}{5} + \frac{4x^3}{3} - 5x\right]\Big|_{-1}^{1}$$

$$= -\pi\left[\frac{1}{5} + \frac{4}{3} - 5\right] + \pi\left[\frac{-1}{5} - \frac{4}{3} + 5\right]$$

$$= \boxed{\pi\left[\frac{-2}{5} \mp \frac{8}{3} + 10\right]}$$

[redacted]

A 10 meter uniform chain with a mass of 5 kilograms per meter is dangling from the roof of a building. How much work is needed to pull the chain up onto the top of the building? (acceleration of gravity: $9.8\frac{m}{s^2}$



mass = 5 kg/m

work

$I\ \Delta h \quad W_i = \text{Force} \cdot \text{Distance}$

$= 5(9.8)\Delta h \quad h_i$

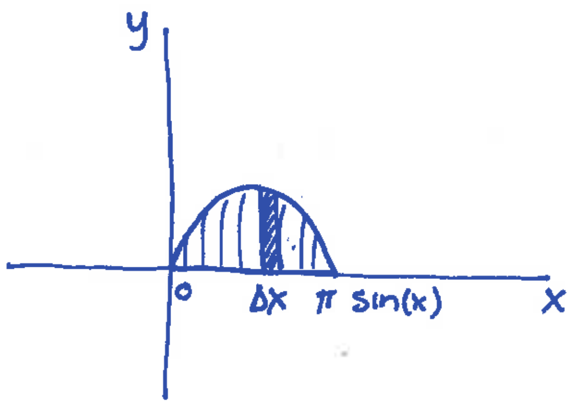
$= 5(9.8) h_i \Delta h.$

$$W_{TOTAL} = 5(9.8)\int_0^{10} h\,dh = 5(9.8)\left.\frac{h^2}{2}\right|_0^{10}$$

$$= 5(9.8)\frac{10^2}{2} = \boxed{\frac{5}{2} \cdot 980\,J}$$

[redacted]

Find the mass of the region bounded by $y = \sin(x)$ and $y = 0$ between $x = 0$ and $x = \pi$, if the density is $\delta(x) = x\frac{g}{cm^3}$.



M_i = Density · Area

$= x_i \sin(x)\, \Delta x$

$$\text{Mass}_{\text{TOTAL}} = \int_0^{\pi} x \sin(x)\, dx$$

let $u = x$ $\quad dv = \sin(x)\,dx$

$du = dx$ $\quad v = -\cos(x)$

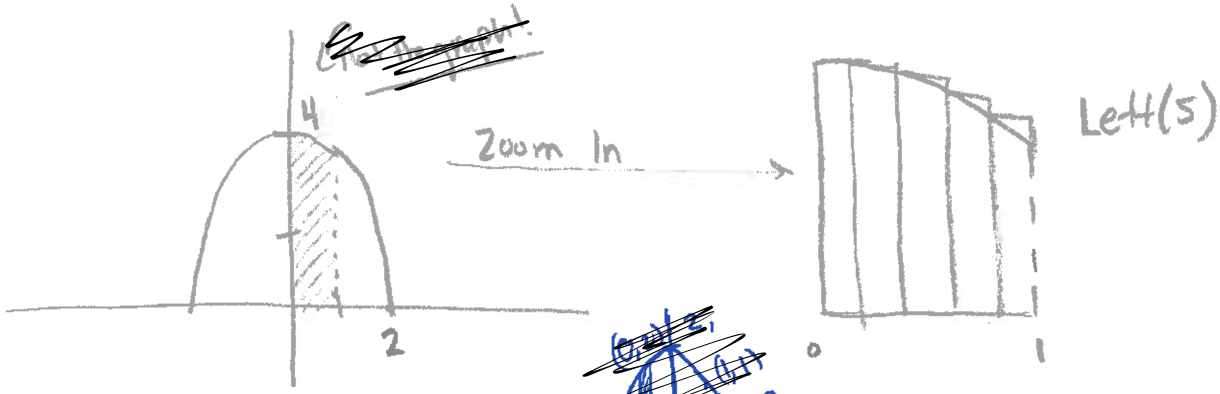
$$= -x\cos(x)\Big|_0^{\pi} + \int_0^{\pi} \cos(x)\,dx$$

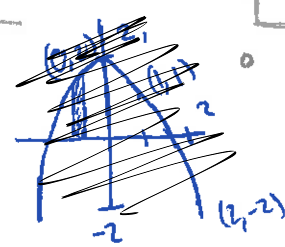
$$= -\pi\cos(\pi) + \sin(x)\Big|_0^{\pi}$$

$$= \pi + \sin(\pi) - \sin(0) = \boxed{\pi \text{ grams.}}$$

[redacted]

Sketch the function $f(x) = -x^2 + 4$, then answer the following questions:





a. How would you use ~~the midpoint rule~~ Left (5) to evaluate $\int_0^1 f(x)\,dx$? Sketch ~~MID~~ Left(5) on your graph of $f(x)$. Divide the region into five sections. Use points from the left to determine height of the box.

b. Would ~~MID~~ Left(5) be an underestimate or overestimate of the value of the integral? It would be an overestimate because the function is decreasing

c. ~~How would you find~~ ~~MID~~ Mid ~~(5)? (Just say in words what you would have to do.)~~
Instead of taking points from the left to determine height, take points from the middle.

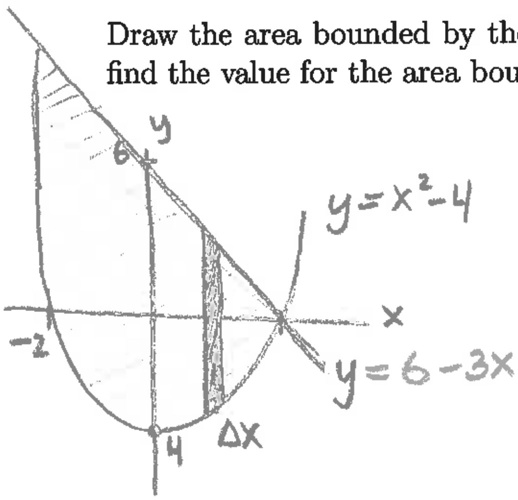
Error: $|E_L| \le \frac{K_1(b-a)^2}{2n}$ — find K_1 — $|f'(x)| = |-2x|$
this is max when $x=1$ so $K_1 = 2$

$= \frac{2(1-0)^2}{2n} = \frac{1}{n} < 10^{-2}$

I need $\boxed{n > 10^2}$

[redacted]

Draw the area bounded by the curves $3x + y = 6$ and $y = x^2 - 4$, then use integration to find the value for the area bounded by the two curves.



Intersections:

$x^2 - 4 = 6 - 3x$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = 2 \quad x = -5$

Take slices vertically.

h_i

Δx

$\text{Area}_i = h_i \Delta x$

$h_i = y_{TOP} - y_{BOTTOM} = [6 - 3x_i] - [x_i^2 - 4]$

$= 10 - 3x_i - x_i^2$

$\text{Area}_i = (10 - 3x_i - x_i^2)\Delta x$

$$\text{Total Area} = \lim_{n\to\infty} \sum_{i=1}^{\infty} (10 - 3x_i - x_i^2)\Delta x = \int_{-5}^{2} 10 - 3x - x^2 \, dx$$

$$= 10x - \frac{3}{2}x^2 - \frac{x^3}{3} \Big|_{-5}^{2}$$

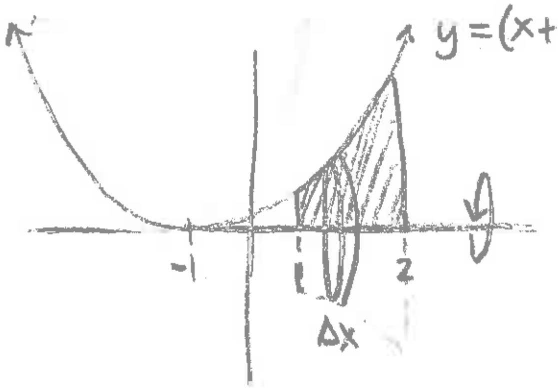
$$= \Big[20 - \underbrace{6}_{12/2} - \frac{8}{3}\Big] - \Big[-50 - \underbrace{\frac{3}{2}25}_{75/2} + \frac{125}{3}\Big]$$

$$= \boxed{70 + \frac{63}{2} - \frac{133}{3}}$$

✓

[redacted]

Find the volume of the solid generated by rotating the region bounded by $y = (x+1)^2$, $y = 0$, $x = 1$, $x = 2$ about the x-axis.



$y=(x+1)^2$ – parabola $y=x^2$ shifted left $(x+1)^2$

Take vertical Slices!

$V_i = \pi r_i^2 \Delta x$

Function − Axis of Rotation

get r_i in terms of x: $r_i = (x_i+1)^2 - 0$

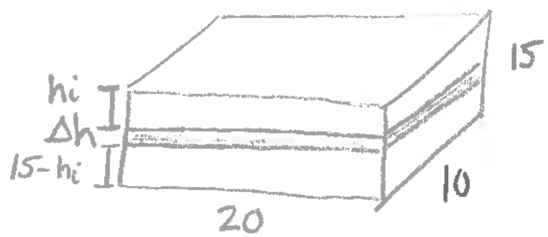
$V_i = \pi (x_i+1)^4 \Delta x$

$\frac{\pi}{5}\left[(x+1)^5\right]_1^2 = \boxed{\frac{\pi}{5}\left[(3)^5-(2)^5\right]}$ ✓

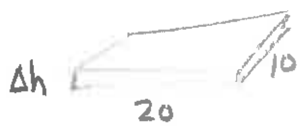
TOTAL VOLUME $= \pi \int_1^2 (x+1)^4\,dx = \frac{\pi}{5}(x+1)^5 \Big|_1^2 = \frac{\pi}{5}\left[243 - 32\right]$?

[redacted]

A rectangular water tank has length 20m, width 10m, and depth 15m. If the tank is full, how much work does it take to pump all the water out? (density of water: $1000\frac{kg}{m^3}$)



Horizontal Slice — Constant Work



$W_i = (\text{Force})(\text{Distance})$

Force: $1000 \, kg/m^3 \cdot \text{Volum} \cdot 9.8$, with Volum $= 200\,\Delta h$

Distance: h_i

$W_i = (9.8)(200)(1000) h_i \Delta h$

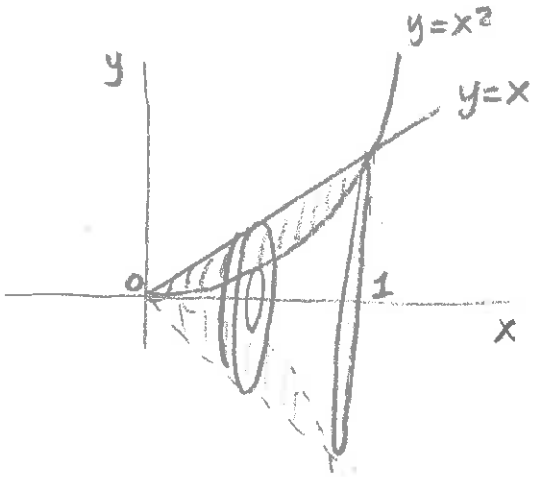
$$\text{TOTAL WORK} = (9.8)(200)(1000)\int_0^{15} h \, dh = \frac{(9.8)(200)(1000)}{2} h^2 \Big|_0^{15}$$

$$= \frac{(9.8)(200)(1000)(15)^2}{2} J$$

$$= \boxed{980000(15)^2 J}$$

[redacted]

Find the mass of the solid created by rotating the region bounded by $y = x$, $y = x^2$ about the x-axis, if the density is $\delta(x) = x + 1 \frac{kg}{m^3}$.



$\rho(x) = x + 1$

$M_i = \text{Mass}_i = \text{Density} \cdot \text{Volume}$

$(x+1)$ $\qquad$ $\pi(r_2^2 - r_1^2)\Delta x$

need r_2 and r_1 in terms of x

$r_2 = x - 0$

$r_1 = x^2 - 0$

$M_i = (x+1)\pi(x^2 - x^4)\Delta x$

$= \pi(x+1)(x^2 - x^4)\Delta x$

$$\text{Total Mass} = \pi \int_0^1 (x+1)(x^2 - x^4)\,dx = \pi \int_0^1 x^3 - x^5 + x^2 - x^4\,dx$$

$$\boxed{= \pi\left[\frac{1}{4} - \frac{1}{6} + \frac{1}{3} - \frac{1}{5}\right]}$$