REVIEW OF CALC I

Average Velocity $= \dfrac{\text{change in position}}{\text{change in time}} = \dfrac{s(b)-s(a)}{b-a}$

Instantaneous Velocity $= \lim\limits_{h \to 0} \dfrac{s(a+h)-s(a)}{h}$ = slope of the position curve

Average rate of Change $= \dfrac{f(a+h)-f(a)}{h}$

Instantaneous rate of Change $= f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h)-f(a)}{h}$

The derivative function $= f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

1. Draw the derivative of a function given its graph
2. Find the derivative using algebra
3. Interpretation of the derivative.

The second derivative $= f''(x) = \lim\limits_{h \to 0} \dfrac{f'(x+h)-f'(x)}{h}$

If $f' > 0$ then f is increasing.
If $f' < 0$ then f is decreasing.
If $f'' > 0$ then f is concave up.
If $f'' < 0$ then f is concave down.

$s(t)$ = position $\quad \dfrac{ds}{dt} = v(t)$ = velocity $\quad \dfrac{dv}{dt} = a(t)$ = acceleration

Derivative Short Cuts! (Should be able to take the derivative of almost anything!)

Constant Multiple

$$\frac{d}{dx} c f(x) = c \frac{df}{dx}$$

Sum and Difference

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{df}{dx} \pm \frac{dg}{dx}$$

Power Rule

$$\frac{d}{dx} x^n = n x^{n-1}$$

Exponential Functions

$$\frac{d}{dx} e^x = e^x \quad \frac{d}{dx} a^x = \ln(a) a^x$$

Product Rule

$$\frac{d}{dx} f(x) \cdot g(x) = f'(x) \cdot g(x) + f(x) g'(x)$$

Quotient Rule

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x) g(x) - g'(x) f(x)}{g(x)^2}$$

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Trigonometric Functions

$$\frac{d}{dx} \sin x = \cos x \quad \frac{d}{dx} \cos x = -\sin x \quad \frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = \sec^2 x$$

Implicit Functions: $x^2 + y^2 = 4$ then $\frac{d}{dx}(x^2 + y^2 = 4)$

$\Rightarrow 2x + 2y \frac{dy}{dx} = 0$ solve for $\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$

chain rule because $y = y(x)$.

Tangent Line Approximation

$f(x) \doteq f(a) + f'(a)(x - a)$ the local linearization of $f(x)$ near $x = a$

Maxima and Minima occur at critical points where $f'(x) = 0$

If we find a point p. such that $f'(p) = 0$
then we can test whether it is a max or min.

First Derivative Test	Second Derivative Test
if f' goes from $+$ to $-$ then p is a maximum	If $f''(p) > 0$ then p is a min
if f' goes from $-$ to $+$ then p is a minimum	if $f''(p) < 0$ then p is a max
	if $f''(p) = 0$ this tells us nothing.

Inflection points: points where a function changes concavity.

<u>For Example</u> $f''(p)=0$ and $f''(p-1)<0$ $f''(p+1)>0$
test points

Global Maximum and Minimum...

1. Find all critical points $f'=0$ classify these
2. Plug critical points back into original function $f(x)$
3. Consider the end points, either plug them into $f(x)$ or take the limit of $f(x)$
4. Choose the point that gives the largest $f(x)$ value as the global max and the smallest $f(x)$ value as the global min.

The Bounds of a function $f(\text{global min}) < f(x) < f(\text{global max})$
always limits y!

- Given information about the max, min, etc draw the function.

★ Applications to Economics: $C(x)$ = cost function
$r(x)$ = revenue function
$\Pi(x) = r(x) - C(x)$ = profit function
maximize profit, fixed costs, costs per product, revenue per product

★ Optimization and Modeling

1. Draw a picture of the story problem ... think physically about the story you are reading
2. Define your variables on the picture.
3. Write down all information... What are you maximizing or minimizing.
4. Find an equation for the thing you are maximizing or minimizing.
5. Get it in terms of just one variable.
6. Now find critical points, max, min etc and answer the question.

Rates and Related Rates.
1. write down all given info
2. Take the derivative using the chain rule, if needed
3. Plug in all given information
4. Solve for the rate you are looking for.

Integration.

$V(t)$	10	20	30
t	1	2	3

Distance traveled: speed x time

$\Delta t = 1$

Left hand sum $= f(1)\Delta t + f(2)\Delta t = 10(1) + 20(1) = 30$
use left data points

Right hand sum $= f(2)\Delta t + f(3)\Delta t = 20(1) + 30(1) = 50$
use right data points

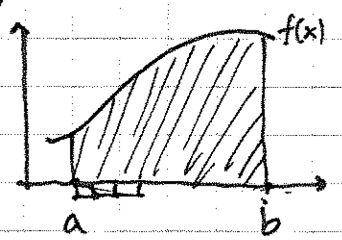
Best estimate is the average of these $\frac{50+30}{2} = 40!$

Riemann Sum to define the definite integral $\Rightarrow \displaystyle\int_a^b f(t)\,dt = \lim_{n\to\infty}\left(\sum_{n=0}^{n-1} f(t_i)\Delta t\right)$

$\Delta t = \frac{b-a}{n}$

could use either sum here!

The Definite integral is just the area under a curve.



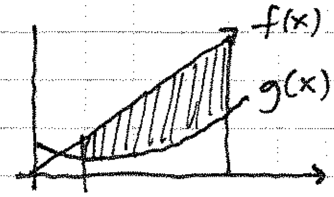
$\displaystyle\int_a^b f(x)\,dx = \text{area}$

FUNDAMENTAL THEOREM

$\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$

where $f(x) = F'(x)$
$F(x)$ is the antiderivative.

Area between two curves



$\displaystyle\int_a^b f(x) - g(x)\,dx$

The definite Integral as an Average $\Rightarrow$ Average value of $f(x)$ from a to b $= \frac{1}{b-a}\int_a^b f(x)\,dx$

Be able to Interpret the integral - what are the units and what is it telling us!

Properties of Integration ↓

Limits $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ || $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

Sums and Constant Mult. $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ || $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

Even and Odd $\int_{-a}^{a} f(x)\,dx = 0$ $f(x) = \text{odd}$ $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ $f(x) = \text{even}$

★ ⟶

Indefinite integral: $\int f(x)\,dx = F(x) + C$ = a family of curves.

Definite integral: $\int_a^b f(x)\,dx = F(b) - F(a)$ = a number.

1. Look at the integrand $f(x)$... what would I take the derivative of to get this

$$\frac{d}{dx}(\quad) = f(x)$$

→ fill in the blank.

this is my $F(x)$

2. Indefinite the solution is $F(x) + C$

definite evaluate $F(x)\Big|_a^b = F(b) - F(a)$

Some rules: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ || $\int e^x\,dx = e^x + C$

$\int \cos x\,dx = \sin x + C$ || $\int \sin x = -\cos x + C$ || $\int \frac{1}{\cos^2 x}\,dx = \tan x + C$

$\int \frac{1}{x}\,dx = \ln|x| + C$ knowing your derivatives really helps here!

Differential Equations and Eqns of Motion...

We know a function that is a rate of change of $f(x)$ and we want a function that $f(x)$

velocity $= v(t)$ $\qquad \frac{ds}{dt} = v(t)$ differential eqn for the position.

uniform acceleration $\qquad \frac{dv}{dt} = a \qquad a =$ constant acceleration

then $\quad v(t) = at + V_0 \qquad V(0) = V_0 =$ initial velocity

$$\frac{ds}{dt} = v(t) = at + V_0$$

$s(t) = \frac{a}{2}t^2 + V_0 t + P_0 \qquad S(0) = P_0 =$ initial position.

1. Write down the differential equation
2. Find the antiderivative, aka integrate both sides
3. Write down the general solution
   - the antiderivative plus a constant
4. Apply initial conditions to solve for the constant, if initial conditions are given.