

MATH 122 - Worksheet Day 24

A **sequence** is an ordered collection of numbers $a_1, a_2, a_3, \dots, a_n$, where each of the a_n 's are a **term** in the sequence with **index** n . We will use the following notation:

$$\{a_n\}_{n=1}^{\infty} = \{a_n\} = a_1, a_2, a_3, \dots$$

Often sequences are defined by a discrete function. For example,

$$a_n = f(n) = \frac{1}{2^n}$$

What is a_1 ? What is a_2 ? What is a_{10} ? What is the fifth term?

It is also useful to be able to write down a **general formula**, given a sequence of numbers.

- Given the sequence 2, 4, 8, 16, 32, 64... I would write down the formula $f(n) = 2^n$.
- Given the sequence -1, 3, -5, 7, -9, 11... I would write down the formula $f(n) = (-1)^n(2n - 1)$.

What does the $(-1)^n$ do in these general formulas? How do we get only odd numbers? How do we get only even numbers?

Write down a general term for the sequence 1, 2, 4, 8, 16, 32...

Sometimes we will define sequences using a **recursive formula**. For example,

$$a_n = a_{n-1} + 3$$

for $n > 1$ and $a_1 = 2$. What are the first six terms in this sequence?

A sequence has **limit** L

$$\lim_{n \rightarrow \infty} a_n = L$$

if $a_n \rightarrow L$ as $n \rightarrow \infty$. This means that as n gets large the terms in the sequence approach L , or we start seeing “almost” the same number as we go from one term to the next.

If

$$\lim_{n \rightarrow \infty} a_n$$

exists (and is equal to a real number), we say that **the sequence** $\{a_n\}$ **converges** (or is convergent). Otherwise, we say that **the sequence diverges** (or is divergent).

PRACTICE PROBLEMS

1. Consider the sequence $\{a_n\}$ defined by

$$a_n = \frac{1}{n}$$

Find $\lim_{n \rightarrow \infty} a_n$. Does the sequence $\{a_n\}$ converge or diverge?

2. Consider the sequence $\{a_n\}$ defined by

$$a_n = n + 2$$

Find $\lim_{n \rightarrow \infty} a_n$. Does the sequence $\{a_n\}$ converge or diverge?

3. Consider the sequence $\{a_n\}$ defined by

$$a_n = (-1)^n$$

Find $\lim_{n \rightarrow \infty} a_n$. Does the sequence $\{a_n\}$ converge or diverge?

4. Consider the sequence $\{a_n\}$ defined by

$$a_n = \left(\frac{1}{4}\right)^n$$

Find $\lim_{n \rightarrow \infty} a_n$. Does the sequence $\{a_n\}$ converge or diverge?

5. Consider the sequence $\{a_n\}$ defined by

$$a_n = (p)^n$$

For what values of p is the sequence convergent?

Make sure you review the rules for limits and remember L'Hopitals rule!