

MATH 122 - Numerical Integration Worksheet

Our goal is to approximate definite integrals using the approximations learned in class: LEFT(n), RIGHT(n), MID(n), TRAP(n), and SIMP(n). **On a separate sheet of paper:**

1. Consider the integral:

$$\int_1^2 \frac{1}{x} dx$$

- a. Evaluate this integral to find the actual value.

- b. Graph $f(x)$ between $1 \leq x \leq 2$, you can use your calculator or desmos.com. Is it increasing or decreasing? Is it concave up or concave down?

- c. Fill in the table of values for $f(x) = \frac{1}{x}$ with the following x values. We can use a spreadsheet to help us!

x	1	1.0625	1.125	1.1875	1.250	1.3125	1.375	1.4375	1.500	1.5625	1.625	1.6875	1.750	1.8125	1.8750	1.9375	2
$f(x)$																	

- d. Using your table of values estimate $\int_1^2 \frac{1}{x} dx$ using the following methods. First identify which points you will use. For example, when calculating LEFT(8) you should use the points $x = 1, 1.125, 1.250, \dots$. Notice that $n = 8$ in each of these cases, in other words you have eight divisions. We calculated sixteen points in the table so that you can do the left, right, and midpoint sums. The points $x = 1.0625, 1.1875, 1.3125, \dots$ will be used to find the midpoint.

- LEFT(8)=
- RIGHT(8)=
- MID(8)=
- TRAP(8)=
- SIMP(8)=

- e. Calculate the error of each of your approximations from part c.

(In part a you calculated the actual value, so use this to calculate the error, actual value minus the estimated value.)

- f. Which methods over estimated? Which methods under estimated? Talk about why this makes sense based on the graph of your function and what you know about how the approximate integration method works.

2. Now we are going to use the estimation methods to approximate an integrals that we cannot solve by hand. Consider the integrals:

$$\int_0^1 \sin(x^2) \, dx$$

$$\int_{-1}^1 e^{-x^2} \, dx$$

- (a) Skip part (a) since we cannot find an exact value.
- (b) Graph the function and describe it.
- (c) Make a table of values for $f(x)$ to be used in your estimation.
- (d) Estimate the integral using each of the methods and $n = 8$.
- (e) Skip part (e), since you don't know the exact value.
- (f) Can you figure out which methods underestimated the value of the integral? Which methods overestimated the value of the integral? (Hint: Think about the shape of the graph compared with what you know about the approximate integration techniques.)