

MATH 122 - Numerical Integration and Error Worksheet

Our goal is to investigate the error in using LEFT(n), RIGHT(n), MID(n), TRAP(n), and SIMP(n) to approximate integrals. **On a separate sheet of paper:**

PART 1: Investigate the error for each of the following integrals.

1.

$$\int_1^2 \frac{1}{x} dx$$

2.

$$\int_0^1 \sin(x^2) dx$$

3.

$$\int_{-1}^1 e^{-x^2} dx$$

4.

$$\int_0^1 e^{x^2} dx$$

- a First plot the integrand using either a calculator or computer over the integration range. Sketch this on your paper. Make sure to label your plot.
- b For each of LEFT(n), RIGHT(n), MID(n), and TRAP(n) draw the approximation on the plot and discuss whether it is an under or over estimate.
- c Now use the error formulas to bound the magnitude of the error for each of LEFT(n), RIGHT(n), MID(n), TRAP(n), **and** SIMP(n).
- d How many subdivisions, n , will you need to approximate the integrals so that $|E| < 10^{-4}$?

GOAL: After investigating the error for the above integrals, try to write down general rules for deciding if the approximation is an over or under estimate. For example, If the integrand is increasing over the range then we know that $LEFT(n) < \int_a^b f(x) dx < RIGHT(n)$. Can you decide what happens for decreasing, concave up, concave down, symmetric, etc...

PART 2: For each of the integrals above answer the following thought questions:

- Imagine that the value of the integral represents the total allowable safe dose of a drug to administer to a patient. Which approximations would you recommend a doctor use to find the value of the integral and why?
- Imagine that the value of the integral represents the minimum dose of a drug that will be effective in treating a disease. Which approximation would you recommend a doctor to use to find the value of the integral and why?
- Imagine that the value of the integral represents the magnitude of earthquake that a building can withstand as a function of the strength of foundation. Which approximation should engineers use to decide how strong the foundation should be?

PART 3: Imagine solving the following integral using approximate integration techniques:

$$\int_0^1 \frac{1}{\sin(x^2)} dx$$

What issues might you run into in approximating the solution? Try doing LEFT(8), what happens? Can you think of ways to approximate the behavior of this integral to find a “solution”?