[redacted] Calc II Day 12

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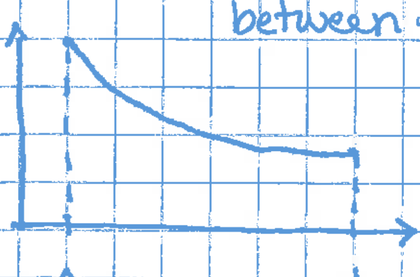
Last time: Area and Volume using integration.

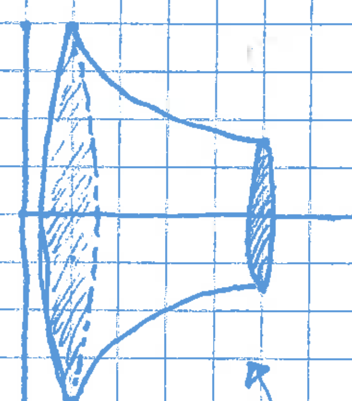
Steps:
1. Draw a Picture and label your region
2. Divide the volume/area into small pieces. Find the volume/area of the small piece
3. Get everything in terms of the small variable Δ
4. Add up all the small pieces — this gives a Riemann sum.
5. Take the limit as $n \to \infty$ giving an integral. The limits of integration are the maximum and minimum values for the variable.

Volumes of Revolution

consider the curve $y = e^{-x}$ between 0 and 1



if we rotate this about the x-axis or about $y=0$ we would get:

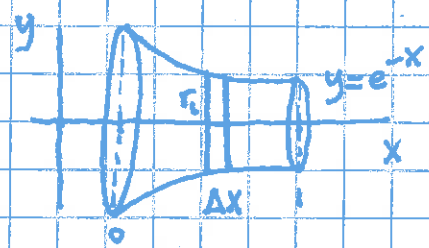


It would trace out a volume

GOAL: Find a numerical value for the volume of this solid.

NOTE: We use the same basic steps as before

1. Draw a Picture



2. Consider a small slice.
- want to go from function to function so slice vertically.

$$\text{Volume of Slice} = \pi r_i^2 \Delta x$$

3. Get everything in terms of x
- where is r_i on our big picture?

Distance from x to our function.

$$r_i = e^{-x_i}$$

$$\text{Volume of Slice} = \pi(e^{-x_i})^2 \Delta x$$

4. Add up all possible slices and take the limit

$$\text{Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} \pi(e^{-x_i})^2 \Delta x$$

5. Find the integral from the limit and evaluate:

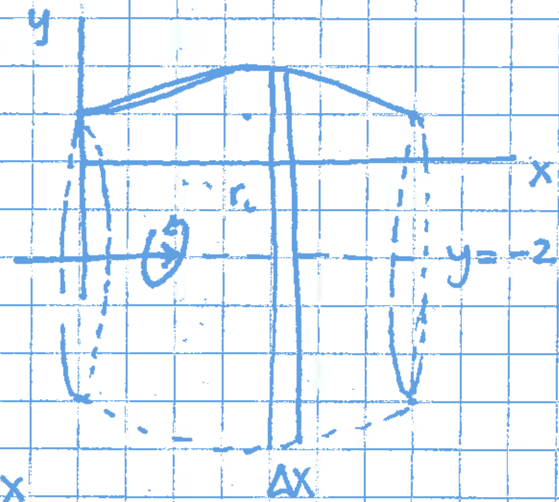
$$\int_0^1 \pi(e^{-x_0})^2\,dx = \pi\int_0^1 e^{-2x}\,dx = -\frac{\pi}{2}e^{-2x}\Big|_0^1 = -\frac{\pi}{2}\left[e^{-2}-1\right]$$

$$= \frac{\pi}{2}\left[1-e^{-2}\right] \sim 1.36.$$

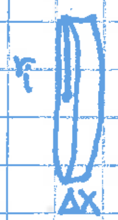
Whenever we are finding volume, area, mass, etc – what kind of answer do we expect?
– what kind of integral should we solve?

Ex Find the volume of the solid created by rotating the function $y = \sin x + 1$ between $x = 0$ and $x = \pi$ about the axis $y = -2$.

1. Draw a picture:



2. Consider the volume of a slice:



$$\text{Volume of Slice} = \pi r_i^2 \Delta x$$

3. Get r_i in terms of x: r_i is the distance between $y=-2$ and $y = \sin x + 1$

$$r_i = \sin x_i + 1 - (-2) = \sin(x_i) + 3.$$

$$\text{Volume of Slice} = \pi(\sin(x_i) + 3)^2 \Delta x$$

4. Sum up all possible slices and take the limit.

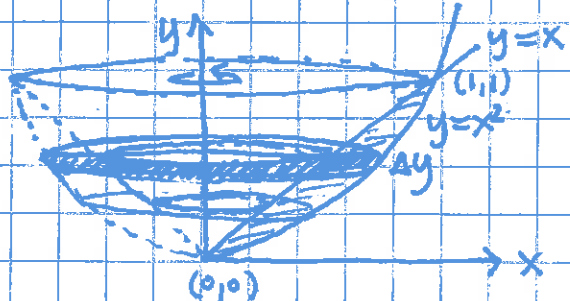
$$\text{Volume} = \lim_{n \to \infty} \sum \pi(\sin(x_i)+3)^2 \Delta x$$

5. Evaluate Integral: $\text{Volume} = \int_0^{\pi} \pi[\sin^2 x + 2\sin x + 9]\, dx$

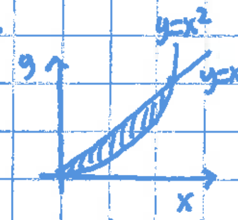
$$= \pi \int_0^{\pi} \left[\frac{1}{2}[1 - \cos 2x] + 2\sin x + 9\right] dx = \frac{\pi}{2}\left[x - \cancel{\frac{1}{2}\sin 2x}^{\,0} - 4\cos x + 18x\right]\Bigg|_0^{\pi}$$

$$= \frac{\pi}{2}\left[(19\pi + 4) - (-4)\right] = \frac{\pi}{2}\left[19\pi + 8\right] = \frac{19\pi^2}{2} + 4\pi$$

Ex Find the volume of the region created by rotating the area bounded by $y=x$ and $y=x^2$ about the y-axis.



1. Draw a picture and find intercepts.

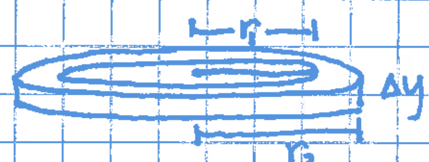


two pics. is better.

looks like a bowl with a cone inside.

2. take a small slice – this time horizontal from function to function.

Always draw your slice →
This helps you to visualize



It is a disk with a hole in the middle.

Volume of Slice = Volume of large disk − Volume of Small disk

$$= \pi r_2^2 \Delta y - \pi r_1^2 \Delta y = \pi(r_2^2 - r_1^2)\Delta y$$

3. Get everything in terms of y: Think of distances.

r_1: goes from 0 to $y=x$
r_2: goes from 0 to $y=x^2$

$r_1 = y_i - 0 \qquad r_2 = \sqrt{y_i} - 0 \qquad$ Volume of Slice $= \pi(y_i - y_i^2)\Delta y$

4. Find the sum of slices: $\lim\limits_{n\to\infty} \sum \pi(y_i - y_i^2)\Delta y$

5. Evaluate Integral: Volume $= \int_0^1 \pi(y-y^2)\,dy$

$$= \pi\left[\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\right]\Big|_0^1 = \pi\left[\tfrac{1}{2} - \tfrac{1}{3}\right] = \boxed{\pi/6}$$

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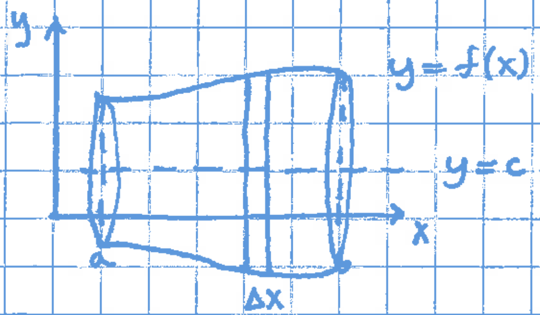
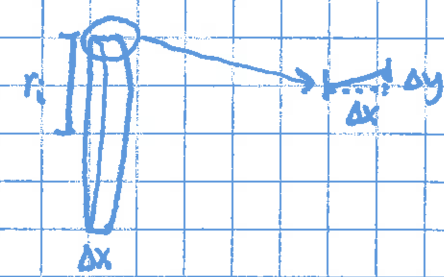
Surface Area for a Volume of revolution.

#1 picture



#2 consider a slice



We are doing two things
1. Length along a curve.
2. Rotating that length around an axis.

Length of the edge: $\sqrt{\Delta x^2 + \Delta y^2}$

Distance traveled around: $2\pi r_i$ circumference.

$$\text{Surface Area of Slice} = 2\pi r_i \sqrt{\Delta x^2 + \Delta y^2}$$

#3 Need all in terms of x: $y = f(x)$ so $\Delta y = f'(x)\Delta x$

and $r_i = f(x) - c$

$$\text{Surface Area of Slice} = 2\pi\left(f(x_i) - c\right)\sqrt{1 + f'(x)^2}\,\Delta x$$

#4 Add em all up: $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} 2\pi\left(f(x_i) - c\right)\sqrt{1 + f'(x)^2}\,\Delta x$

#5 Integrate

$$\text{Surface Area} = 2\pi\int_a^b \left(f(x) - c\right)\sqrt{1 + f'(x)^2}\,dx$$

★ ★ almost always integrate this on Maple !! ★ ★
OR Wolfram Alpha