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8.5 Applications of Integration: Physics

<u>Work = Force · Distance</u> "exerting a force in the direction of motion"

- Walking while holding a book @ constant height – no work done on book
- Lifting the book off the desk – work done.

<u>Ex</u> Find the work done on an object when a force of 2 Newtons moves it 12 meters.

$F = F \cdot d = 2 \cdot 12 = 24 \text{ N}\cdot\text{m} = 24 \text{ Joules}.$

↳ the force was constant ⇒ Easy Case

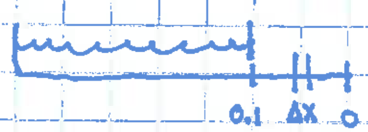
<u>Ex</u> Hooke's Law: The Force required to compress a spring x-meters is given by

$F = +kx$

$k =$ spring constant

Find the work done in compressing the spring 0.1 meters if $k = 8 \text{ N/m}$

draw picture



consider the work to move the spring only a small distance.

$W = F \cdot d = kx_i \Delta x = 8x_i \Delta x$

everything in terms of x ✓

$$\lim_{n \to \infty} \sum_{i=1}^{n} 8x_i \Delta x = \int_0^{.1} 8x\,dx = 4x^2 \Big|_0^{.1} = 0.04 \text{ J}$$

In General: $W = \int_a^b F(x)\,dx$ $\quad$ $F(x)$ = force moving an object from a to b.

Force due to Gravity

mass – amount of a substance – measured in kg.

weight – force exerted on an object by gravity – measured in pounds.

ON THE MOON – you have the same mass but a different gravitational pull means you have a different weight.

Ex A. Work done moving a 5 pound book 3 feet off the floor:

$$W = F \cdot d = 5 \cdot 3 = 15 \text{ foot} \cdot \text{pounds} = 15 \text{ ft lbs.}$$

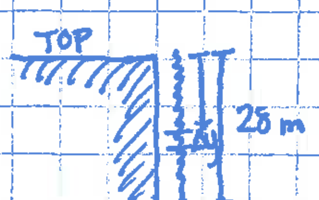
B. Work done lifting a 1.5 kg book 2 meters off the floor:

$$W = F \cdot d = (1.5)(9.8)(2) = 29.4\,J$$

multiply by $g = 9.8\ m/s^2$

Ex Now try one where the force depends on height...

A 28 m uinform chain with a mass of 2 kg per meter is hanging from the roof of a building.
How much work is done in pulling the chain up to the top of the building?



Problem: Some of the chain moves only a short distance ... once its @ the top we are no longer working to pull it up!

$$\frac{\text{Force}}{\text{meter}} = \frac{\text{mass}}{\text{meter}} \cdot g = (2)(9.8) = 19.6\ N/m$$

Consider a small slice Δy
hanging at a distance y_i below the top.

Work on Piece $= F \cdot d = 19.6\,\Delta y \cdot y_i = 19.6\, y_i\,\Delta y$

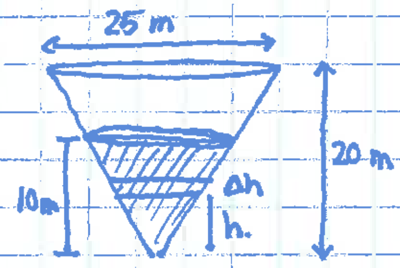
$$\lim_{n\to\infty} \sum_{i=1}^{n} 19.6\, y_i\,\Delta y = \int_0^{28} 19.6\, y\, dy = 9.8\, y^2 \Big|_0^{28} = 7683.2\,J$$

WE ALWAYS TRY TO UNDERSTAND A SLICE FIRST !!

EX Calculate the work done in pumping the oil from a cone shaped tank out to the rim.

Assume: $\delta(x) = 800\ kg/m^3$ Depth of Oil $= 10$ m.

#1 picture



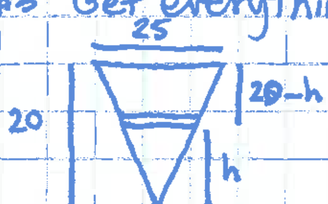
#2 Consider a small slice – how much work to move the slice?

Work $= F \cdot d = $ Mass $\cdot g \cdot d$
$= \delta(x) \cdot$ Area $\cdot g \cdot d$

Work on slice: $800 \cdot \pi r_i^2 \Delta h \cdot 9.8 \cdot (20 - h_i)$

$$W = (800)\cdot(9.8)\pi\, r_i^2 (20 - h_i)\Delta h$$

#3 Get everything in terms of h.
radius is a function of height...



$$\frac{20}{25} = \frac{h}{2r} \qquad 2r = \frac{25h}{20} = \frac{5h}{4} \qquad r = \frac{5h}{8}$$

Work on Slice $= [800 \cdot 9.8\pi]\left(\frac{5h_i}{8}\right)^2 (20 - h_i)\Delta h$

$$\lim_{n\to\infty} \sum_{i=1}^{n} 800 \cdot 9.8\pi \left(\frac{5h_i}{8}\right)^2 (20 - h_i)\Delta h = 800\left(\frac{5}{8}\right)^2 \pi (9.8) \int_0^{10} h^2 (20 - h)\, dh$$

$\sim 4.0 \times 10^7\ J$

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8.2 Fluid Pressure and Force.

Fluid Force is the force on an object that is under water.

- You dive to the bottom of the pool you feel pressure.

$$\text{Pressure} = \left(\text{density of fluid}\right)\left(\text{acceleration of gravity}\right)\left(\text{depth}\right) \qquad \frac{\text{Newtons}}{\text{meter}^2}$$

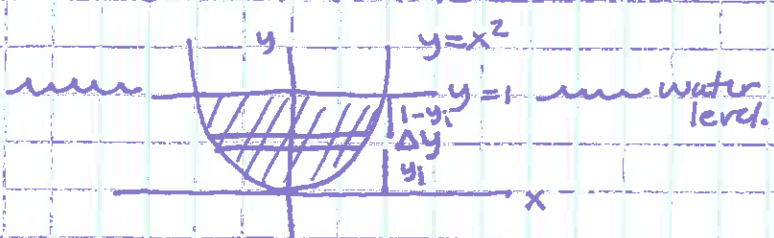
diving in lava would have more pressure than diving in air

on the moon we could go deeper w/ less pressure "The weight of the water is less"

$$\text{Force from a Fluid} = (\text{Pressure})(\text{Area}) \qquad \text{Newtons}$$

<u>Ex</u> A plate, bounded by the parabola $y=x^2$ and the line $y=1$, is submerged in water, with the top of the plate at the surface. What is the force on the plate from the fluid.

Draw the Plate



Consider a slice at constant depth:

Force = Pressure × Area

$1000\ \frac{kg}{m^3} \times 9.8 \times (1-y_i)$

$\to w_i\,\Delta y$

$$F_i = 9800(1-y_i)\,w_i\,\Delta y$$

Now get w_i in terms of y_i

$$w_i = x_R - x_L$$

$$x_R = \sqrt{y_i} \quad x_L = -\sqrt{y_i}$$

$$w_i = \sqrt{y_i} - (-\sqrt{y_i}) = 2\sqrt{y_i}$$

$$F_i = 9800(1-y_i)2\sqrt{y_i}\,\Delta y$$

Now we can add up the pieces...

$$\text{Total Force} = 2(9800)\int_0^1 (1-y)\sqrt{y}\,dy$$

$$= 19600\left[\tfrac{2}{3}y^{3/2} - \tfrac{2}{5}y^{5/2}\right]\Big|_0^1$$

$$= 19600\left(\frac{2}{3} - \frac{2}{5}\right) = 19600\left(\frac{10-6}{15}\right) = 19600\left(\frac{4}{15}\right)$$

$$\cong 5226.67 \text{ N}$$