8.2 Fluid Pressure and Force.

Fluid Force is the force on an object that is under water.

- You dive to the bottom of the pool you feel pressure.

$$\text{Pressure} = \underbrace{\left(\text{density of fluid}\right)\left(\text{acceleration of gravity}\right)}_{\text{weight per } m^3}\left(\text{depth}\right) \quad \frac{\text{Newtons}}{\text{meter}^2}$$

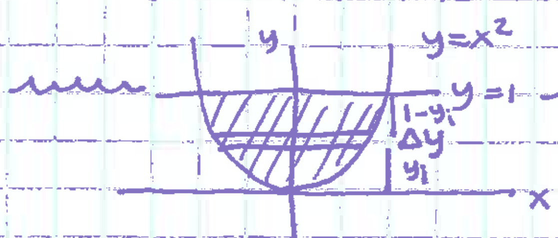
diving in lava would have more pressure than diving in air

on the moon we could go deeper w/ less pressure "The weight of the water is less"

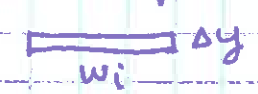
$$\text{Force from a Fluid} = (\text{Pressure})(\text{Area}) \quad \text{Newtons}$$

<u>Ex</u> A plate, bounded by the parabola $y=x^2$ and the line $y=1$, is submerged in water, with the top of the plate at the surface. What is the Force on the plate from the fluid.

Draw the Plate



Consider a slice at constant depth:



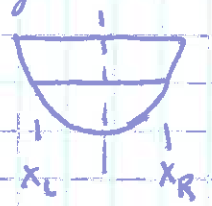
Force = Pressure $\times$ Area

Area $\rightarrow w_i \Delta y$

Pressure $\rightarrow 1000 \frac{kg}{m^3} \times 9.8 \times (1-y_i)$

$F_i = 9800(1-y_i)\, w_i\, \Delta y$

Now get w_i in terms of y_i



$$w_i = x_R - x_L$$

$$x_R = \sqrt{y_i} \qquad x_L = -\sqrt{y_i}$$

$$w_i = \sqrt{y_i} - (-\sqrt{y_i}) = 2\sqrt{y_i}$$

$$F_i = 9800(1-y_i)\,2\sqrt{y_i}\,\Delta y$$

Now we can add up the pieces...

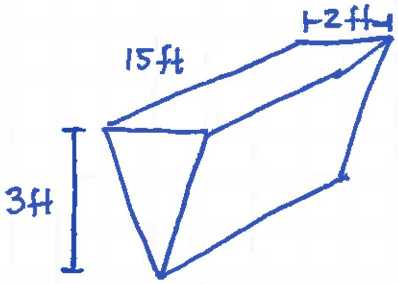
$$\text{Total Force} = 2(9800)\int_0^1 (1-y)\sqrt{y}\,dy$$

$$= 19600\left[\frac{2}{3}y^{3/2} - \frac{2}{5}y^{5/2}\right]\Bigg|_0^1$$

$$= 19600\left(\frac{2}{3} - \frac{2}{5}\right) = 19600\left(\frac{10-6}{15}\right) = 19600\left(\frac{4}{15}\right)$$

$$\cong 5226.67 \text{ N}$$

Ex Consider the triangular tank:

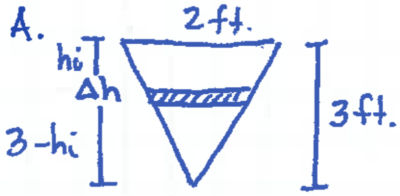


A. If the tank is full of water, find the force on the triangular side.

B. Then calculate the work to pump the water out of the tank.

Density of water: $1000\ kg/m^3$ OR $62.4\ lb/ft^3$ ✓

A.



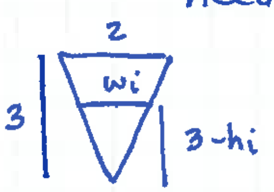
Consider a slice of constant depth (Pressure)

F_i = Pressure · Area

= Density · Depth · Area

$= 62.4 \cdot h_i \cdot w_i \Delta h$ lb. ✓

need w_i in terms of h



$$\frac{3}{2} = \frac{w_i}{3-h_i} \quad \text{so} \quad w_i = \frac{2}{3}(3-h_i)$$

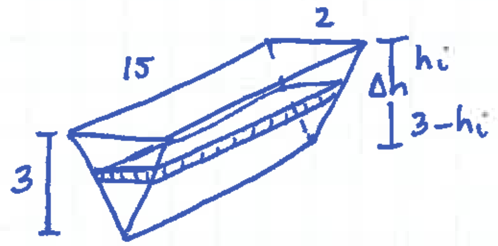
$$F_i = 62.4\, h_i \left(\tfrac{2}{3}(3-h_i)\right) \Delta h$$

$$= \frac{2}{3} \cdot 62.4\ h_i (3-h_i) \Delta h$$

$$\text{Total Pressure} = \frac{2}{3}(62.4)\int_0^3 h(3-h)\,dh = \frac{2}{3}(62.4)\int_0^3 3h - h^2\,dh$$

$$= \frac{2}{3}(62.4)\left[\frac{3}{2}h^2 - \frac{h^3}{3}\right]_0^3 = \frac{2}{3}(62.4)\left[\frac{27}{2} - \frac{27}{3}\right] = 421.2 \text{ lb.}$$

B.



Consider a slice
of constant work

- the long sides are always 15 length.
- the width w_i changes based on height.

$$W_i = \text{Force} \cdot \text{Distance}$$
$$= 62.4 \cdot \text{Volume} \cdot \text{Distance}$$
$$= 62.4 \cdot 15\, w_i\, \Delta h \cdot h_i$$

then need w_i in terms of h_i

$w_i = \frac{3}{2}(3-h_i)$ From part A.

$$W_i = (62.4)(15)\tfrac{3}{2}(3-h_i)h_i\,\Delta h.$$

Add up The slices

$$W_{TOTAL} = (62.4)(15)\tfrac{3}{2}\int_0^3 h(3-h)\,dh = (62.4)(15)\tfrac{3}{2}\int_0^3 3h - h^2\,dh$$

$$= (62.4)(15)\left(\tfrac{3}{2}\right)\left[\frac{3h^2}{2} - \frac{h^3}{3}\right]\Bigg|_0^3 = (62.4)(15)\left(\tfrac{3}{2}\right)\left[\frac{27}{2} - \frac{27}{3}\right]$$

$$= (15)(62.4)\left(\tfrac{3}{2}\right)\left[\frac{27}{2} - \frac{27}{3}\right] = 6318 \text{ J}$$