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Transforming The Integrand — algebra techniques.

Factoring / Partial Fractions
Long Division
Complete the square

use these to simplify and integrate or to transform to table integrals.

<u>Factoring</u> <u>Ex</u> $\int \frac{3x+7}{x^2+6x+8} =$ • use when you have a polynomial in denominator

$$(x^2+6x+8) = (x+2)(x+4)$$

so $= \int \frac{3x+7}{(x+2)(x+4)}\,dx$ looks like # on the tables.

By Hand... Need <u>Partial Fraction Decomposition</u> (aka break this into nice pieces)

★. Use when the order of denominator is greater than the order of the numerator.
<u>GOAL</u> Break up factored denominator into pieces.

1. Write down integrand and what you want.

$$\frac{3x+7}{(x+2)(x+4)} = \frac{A}{(x+2)} + \frac{B}{(x+4)}$$

I want seperate fractions.

2. Multiply through by the LHS denominator

$$3x + 7 = A(x+4) + B(x+2)$$

3. Gather terms on the RHS.

$$3x + 7 = (A+B)x + (4A+2B)$$

4. Now Ballance the equation to get two equations for A and B

$3 = (A+B)$ on either side these have x's next to them

$7 = 4A + 2B$ on either side these are the constants.

5. Solve for A and B.

$$B = 3 - A \rightarrow 7 = 4A + 6 - 2A = 2A + 6$$

$$2A = 1 \quad A = 1/2$$

$$B = 3 - 1/2 = 5/2$$

6. Sub these: $A = 1/2$ and $B = 5/2$ back into what you wrote down in part 1

$$\frac{3x+7}{(x+2)(x+4)} = \frac{1/2}{(x+2)} + \frac{5/2}{(x+4)}$$

7. Evaluate the integral

$$\int \frac{1/2}{(x+2)}\,dx + \int \frac{5/2}{(x+4)}\,dx = 1/2 \ln|x+2| + 5/2 \ln|x+4| + c.$$

# Polynomial Long Division

★ Use when the order of the numerator is greater than ~~or same as~~ the order of denominator.

or same as

Ex $\int \frac{x^2}{x^2+4}\,dx$ use long division.

1. Rewrite the integrand:

$$\begin{array}{r|l} & 1 \\ x^2+4 & x^2 \\ & \underline{x^2+4} \\ & -4 \end{array}$$ remainder

$$\frac{x^2}{x^2+4} = 1 - \frac{4}{x^2+4}$$

2. Sub into integral $\int 1\,dx - \int \frac{4}{x^2+4}\,dx$

this looks just like table # $\int \frac{1}{x^2+a^2}\,dx$

later we will learn trigonometric substitutions to solve these by hand.

## COMPLETE THE SQUARE

✫ Use when the order of the denominator is greater than the order of the numerator AND you cant factor.

Ex $\int \frac{1}{x^2+6x+14}\,dx$

cant factor this denominator...

1. Take denominator and write down what you want

$$x^2+6x+14 = (ax+b)^2 + c$$ a perfect square plus a constant.

2. Expand the right hand side

$$x^2+6x+14 = a^2x^2+2abx+b^2+c$$

3. Ballance the two sides by equating like powers of x. and solve for a, b and c.

$1 = a^2$ so $a=1$

$2ab=6 \rightarrow 2b=6$ so $b=3$

$b^2+c=14 \rightarrow 9+c=14$ so $c=5$

4. Sub back into your "wish" in part 1.

$$x^2+6x+14 = (x+3)^2+5$$

5. Use this in the integral

$$\int \frac{1}{(x+3)^2+5}\,dx = \int \frac{1}{w^2+5}\,dx$$

looks like a table integral #

let $w = x+3$

$dw = dx$

**Worksheet**

Special Cases of Partial Fractions.

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{Bx}{(x+1)^2}$$ Repeated Roots.

$$\frac{2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$ Order of Denom.

$$\frac{5x}{(x^3-1)(x+1)} = \frac{A}{(x+1)} + \frac{Bx^2+Cx+D}{(x^3-1)}$$ Numerator doesn't effect guess.