CALC II DAY 12

7.7 : Improper Integrals.

Two Basic Types:
1. Limits of integration infinite
2. Integrand becomes infinite.

Ex $\displaystyle\int_1^{\infty} \frac{1}{x^2}\,dx$

- sometimes we want our variable $\to$ Infinity
- How do we deal with this?

1. Write down the integrals that you CAN solve

$$\int_1^{b} \frac{1}{x^2}\,dx \qquad b = \text{a number}$$

2. Solve that integral

$$= -\frac{1}{x}\Big|_1^{b} = -\frac{1}{b} + 1$$

3. Take the limit as b goes to ∞.

$$\lim_{b\to\infty}\left[-\frac{1}{b}+1\right] = 1$$

Review of limits
Ch. 1.8 p. 48

★ If $\displaystyle\lim_{b\to\infty}\left[\int_a^b f(x)\,dx\right]$ is a finite number then we

say $\displaystyle\int_a^{\infty} f(x)\,dx$ <u>CONVERGES</u> and $\displaystyle\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$

Otherwise, we say that $\displaystyle\int_a^{\infty} f(x)\,dx$ ~~[illegible]~~. <u>DIVERGES</u>

This idea raises some questions ...

How can a curve that goes to infinity on one side have finite area?

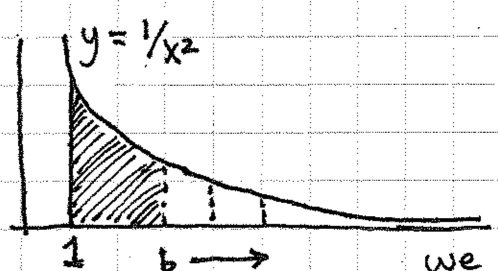
$$\int_1^b \frac{1}{x^2}\,dx = -\frac{1}{b} + 1$$

$b=10 \quad -\frac{1}{10}+1 = 0.9$

$b=100 \quad -\frac{1}{100}+1 = 0.99$

$b=1000 \quad -\frac{1}{1000}+1 = 0.999$

Increasing b gets us closer to 1.



we are adding smaller and smaller areas!

When would this ever apply to the real world?

- Say we wanted to estimate the mass of the earths atmosphere ...
  - we know where it starts
  - where does it end?

<u>Ex</u> Find the amount of energy it takes to separate two hydrogen atoms each with one electron and one proton and opposite charge 1.6×10^{-19}.

Assume an initial distance $R_B = 5.3 \times 10^{-11}$ m The Bohr radius.

Energy to Separate two Charged Particles $\longrightarrow$ $E = \int_a^b \frac{kq_1q_2}{r^2}\,dr$

q_1 and q_2 magnitude of charges

$k = 9 \times 10^9$ a constant.

$$E = \int_{R_B}^{\infty} \frac{kq_1q_2}{r^2}\,dr = kq_1q_2 \int_{R_B}^{\infty} \frac{1}{r^2}\,dr$$

we just did this integral ...

$$\lim_{b\to\infty} \int_{R_B}^{b} \frac{1}{r^2}\,dr = \lim_{b\to\infty} \left[\frac{-1}{r} \Big|_{R_B}^{b} \right] = \lim_{b\to\infty} \left[-\frac{1}{b} + \frac{1}{R_B} \right]$$

so $E = \dfrac{kq_1q_2}{R_B} \sim 4.35 \times 10^{-18}\,\text{J}$

about the same amt of energy required to lift a speck of dust off the ground.

<u>Ex</u> $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$

1. Write down an integral that you CAN solve

$$\int_1^{b} \frac{1}{\sqrt{x}}\,dx =$$

2. Solve it : $= 2x^{1/2}\Big|_1^{b} = 2b^{1/2} - 2$

3. Take the limit.

$$\lim_{b\to\infty} \left[2b^{1/2} - 2 \right] = \infty$$

The integral DIVERGES !

<u>Ex</u> <u>You Try</u> $\displaystyle\int_0^{\infty} e^{-5x}\,dx.$

"The integral CONVERGES TO 1/5."

NOTE. $$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

$$= \lim_{a\to-\infty}\left[\int_a^c f(x)\,dx\right] + \lim_{b\to\infty}\left[\int_c^b f(x)\,dx\right]$$

- c is arbitrary ... it will cancel properly in the end.
- if either of these diverge then the whole thing diverges.

<u>EX</u> $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ Problem here → the integrand is undefined @ $x=0$!

1. Take out the problem point

$$\int_a^1 \frac{1}{\sqrt{x}}\,dx =$$

2. Solve this integral: $= 2\sqrt{x}\Big|_a^1 = 2-2\sqrt{a}$

3. Take the limit as you approach the problem point

$$\lim_{a\to 0}\left[2-2\sqrt{a}\right] = 2$$ The integral CONVERGES to 2.

Ex $\int_0^2 \frac{1}{(x-2)^2}\,dx$ Where is the problem? Evaluate this...

$$\lim_{b\to 2}\left[\frac{-1}{b-2} - \frac{1}{2}\right] = -\infty$$ undefined

The integral diverges.