Math for Data Science

Normal Distribution Testing

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Important Information

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- Office Hours take place in Duke 209 unless otherwise noted – Office Hours Schedule

Today's Goals:

- Normal Distribution
- Hypothesis Testing
- F-test and t-test on Polynomial Regressions

Probabilities so Far

We have discussed how to use data to compute probabilities and how to combine probabilities to ask more interesting questions (Joint, Union, Conditional). We have also explored the idea of probability distributions and looked at a few examples where they might arise in our data.

Today we will talk about a distribution that many of you have heard of **The Normal Distribution** and discuss the idea of **Hypothesis Testing** in general terms.

Finally, we will return to our old friend polynomial regression, and explore a few more tests that can give us information about the probability that we have a good fit for our given data.

The Normal Distribution

First, I am assuming that you are somewhat familiar with the ideas of the mean and standard deviation of data. In python we can use:

```
np.mean()
np.std()
```

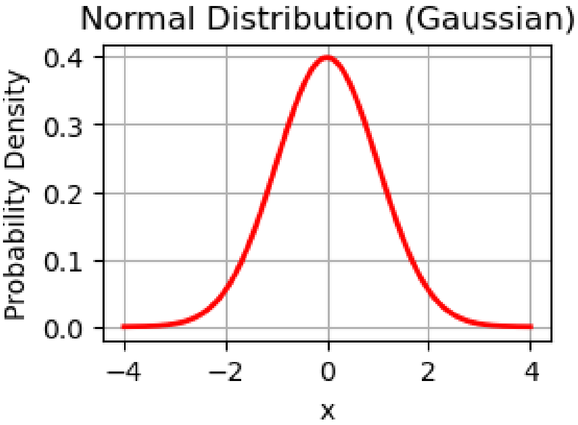
To calculate the mean and standard deviation of our data.

The Normal Distribution

The **Normal Distribution** also called the **Gaussian Distribution** is a symmetry bell shaped curve that has most of it's mass around the mean.

Here is a plot of the Standard Normal Distribtion, where the mean is $\mu = 0$ and the standard deviation $\sigma = 1$.

The Normal Distribution



Properties of a Normal Distribution

The normal distribution arises often in nature. Think about things like the height or weight of animals. Some might be really large or really small, but most are somewhere in the middle.

Properties of a Normal Distribution

- It is symmetrical, mirrored across the mean value (the center)
- Most of the mass is at the center
- It has spread (width) based on the standard deviation
- The "tails" on either side of the mean approach zero as $x \to \pm\infty$

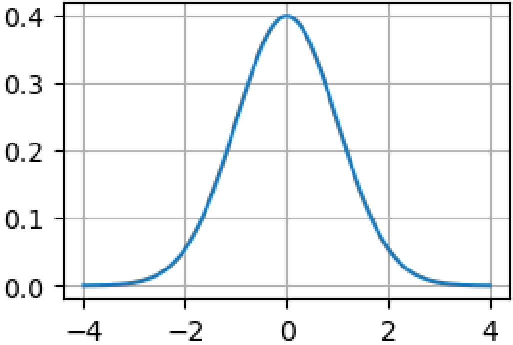
Properties of a Normal Distribution

The probability density function for the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

We can actually plot this function and see that it is the same as what we get from norm.pdf() above.

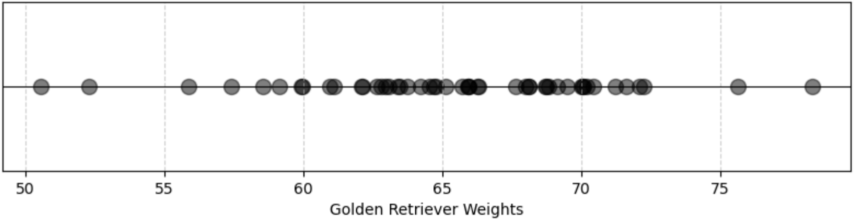
Properties of a Normal Distribution



Example

*From our book**

Let's say we sample the weight of 50 adult golden retrievers and plot them on a number line.



Here we see that there are more values toward the center, around 50-70 pounds than at the edges. We could also plot this data on a histogram!

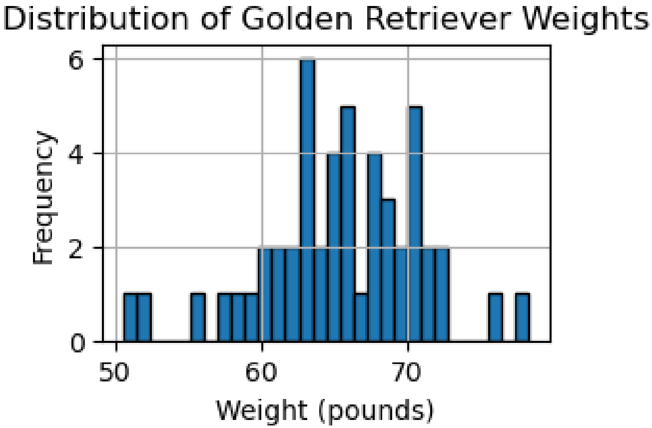
Histogram

Remember that a histogram is a bar graph that helps you visualize the patterns and trends within a set of data by grouping values into ranges and showing how often those ranges occur.

- The width of the bars is set by the number of bins
- The height of the bars is set by the frequency of observations

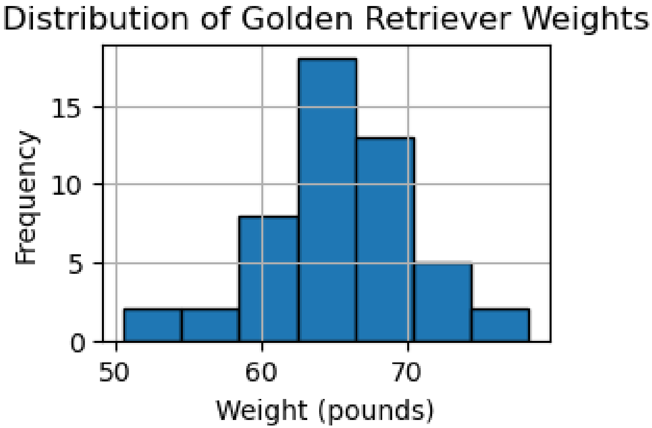
It is up to you to choose the number of bins! Below you will see code for 30 bins and for 7 bins. Which one helps us to see the bell curve that we expect from normally distributed data?

Histogram



Distribution of Golden Retriever Weights
Frequency
0
2
4
6
50
60
70
Weight (pounds)

Histogram


Distribution of Golden Retriever Weights
Frequency
15
10
5
0
50
60
70
Weight (pounds)

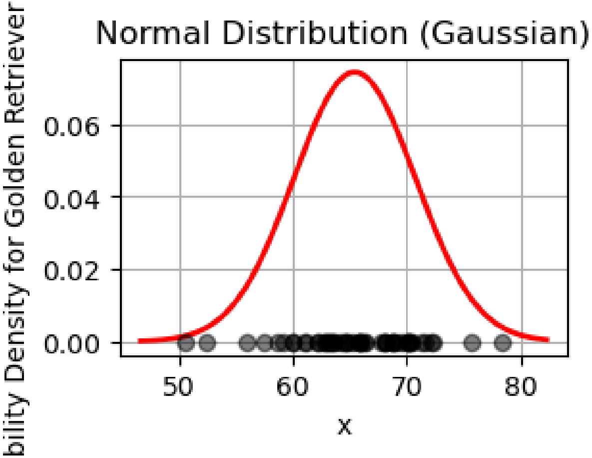
Histogram

As we decrease our bins we start to see that our data does begin to fill in a bell curve (not perfect!) I don't expect it to be perfect because the data is just a sample of the full population.

Let's calculate the mean and the standard deviation of our data and compare to a normal distribution of this data.

Normal Distribution

```
The average is: 65.405043722571
The standard deviation is: 5.3610122946938406
```



Cumulative Distribution Function

Now we are in the position to ask some questions!

1. What is the probability that a random golden retriever will have a weight between 60 and 70 pounds?

Remember **The normal distribution is a continuous distribution** so we need to use the area under the curve to understand probabilities! We can use the Cumulative Distribution Function (CDF) to answer the question.

- Find the probability that the weight is between 0 and 70
- Find the probability that the weight is between 0 and 60
- Subtract the second calculation from the first calculation to get the probability that the weights are between these two numbers

```
The probability that the weight is between 60 and 70 is 0.6476
```

You Try

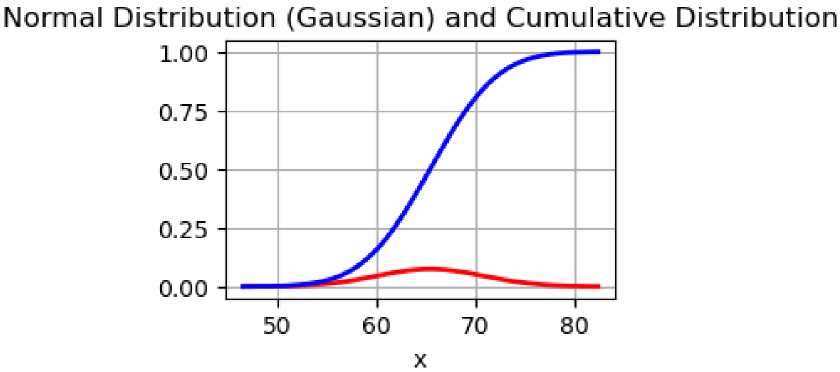
1. What is the probability that the weight is between 50 and 60?
2. What is the probability that the weight is less than or equal to 50?
3. How do these numbers compare to the probability of being between 60 and 70?
4. Does this make sense? Why or why not?

Inverse CDF

In the example above we calculated the probability that we observe weights within a range from a random sample of the population. AKA: what is the probability the weight is between A and B? Sometimes we want to ask a different question. What is the weight that a certain percentage of the population will fall under?

Let's start by plotting the CDF - this is a plot of the area under the probability distribution function as we increase the x-value or the weight.

PDF and CDF


Normal Distribution (Gaussian) and Cumulative Distribution
1.00
0.75
0.50
0.25
0.00
50
60
70
80
x

Inverse CDF

Some things to notice:

- as x increases the CDF increases - there is more area under the curve as we increase the x-value.
- when $x = 1$ the CDF $= 1$ - the total area under the curve is 1.

How can we use this picture? Given an x-value (a weight) we can see what the area under the curve is by looking it up on the blue line. For example, if we want the probability that a golden retrievers weight is below 65 points, we can see that this corresponds to a value of about 0.45.

Inverse CDF

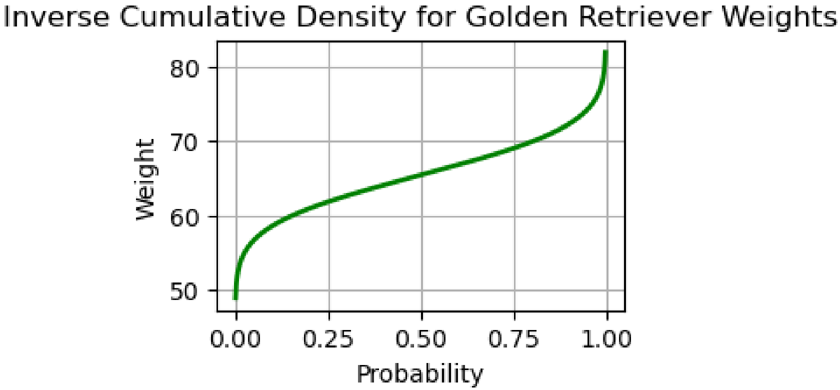
But now what if I wanted to turn this question around? For example, what weight should I expect 90% of my population to be below? This is going backward - we loo up the percentage on the y-axis and return the weight from the x-axis. **This is an inverse!**

The **inverse cumulative distribution function** also called the **percent point function** can be both plotted and calculated using python

```
72.27545742173973
```

We expect 90% of our population to have a weight below 72.28 pounds.

Inverse CDF


Inverse Cumulative Density for Golden Retriever Weights
Weight
80
70
60
50
0.00
0.25
0.50
0.75
1.00
Probability

You Try

- What weight should I expect 20% of my population to be below?
- What weight should I expect half of my population to be below and half to be above?
- What weight should I expect 60% of my population to be above?

Is my data normal?

When dealing with a normal distribution, the "68-95-99.7 rule," also known as the empirical rule, provides a useful guideline for understanding the percentage of data that falls within certain standard deviations of the mean. Here's a breakdown:

- Within 1 standard deviation: Approximately 68% of the data falls within one standard deviation of the mean.
- Within 2 standard deviations: Approximately 95% of the data falls within two standard deviations of the mean. - Within 3 standard deviations: Approximately 99.7% of the data falls within three standard deviations of the mean.

One way to test for outliers is to look for how many scores are outside of the 3-standard deviation range.

Exploring Normal Distributions - You Try

Below is code that generates sample data for two groups of students: Cheaters and Non-Cheaters. You can run the cell below without making changes the first time to see what happens if there are no cheaters:

```
num_students = 100
num_cheaters = 0
```

Exploring Normal Distributions - You Try

PART 1:

1. Plot a Histogram of the data when there are no cheaters and talk about the mean and standard deviation of the data.
2. What is the probability the a students score is 75% or below?
3. What is the probability that a students score is above 90%?
4. What score should 75% of the class score below?
5. What score should 90% of the class score below?
6. What is the score for the top 10% of students?

Exploring Normal Distributions - You Try

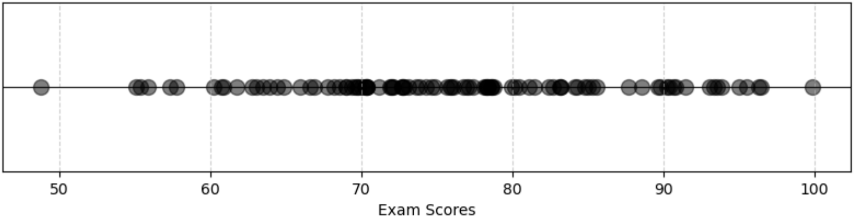
PART 2:

1. Now redo the experiment but now add some cheaters:

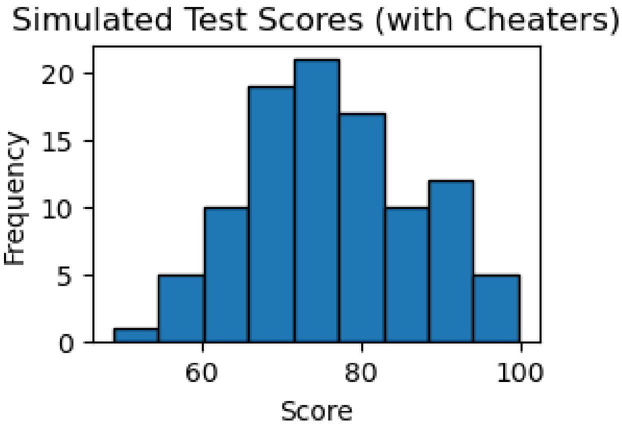
```
num_students = 90
num_cheaters = 10
```

2. What do you notice about your distribution. Say in words what happens to the mean and standard deviation. Talk about what happens to your histogram. How do your probabilities and percent populations change?

Exploring Normal Distributions - You Try



Exploring Normal Distributions - You Try




```
Mean of all scores: 76.0361305225264
Standard deviation of all scores: 10.753250301980355
Probability below 75: 0.4616192750423362
```

Check if the data is 'Normal'

Here is code to help us check if our data really is following a normal distribution:

- Run the 68-95-99.7 rule code below for the case of cheaters and no cheaters.
- Can you figure out what the code is doing?
- You should start to see some skew in the data - the data is not fitting the normal distribution quite as well.

```
Within 1 standard deviations we have 66.0 percent of the data
Within 2 standard deviations we have 98.0 percent of the data
Within 3 standard deviations we have 100.0 percent of the data
```

Uniformly Distributed data

We are going to do an interesting experiment. Below you will find code that simulates the rolling of dice. You can roll as many dice as you want, from 1 up. The code then counts up the total score you got while rolling. We will start by just rolling 1 die:

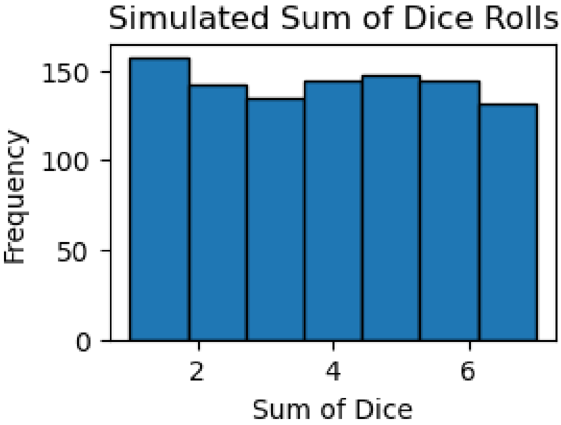
```
num_rolls = 1000
num_dice = 1
```

Before you run the code What do you expect to see.... How many 1's, 2's, 3's etc?

Uniformly Distributed data

The mean is: 3.938

The standard deviation is: 2.00353587439806



You Try

Now we will explore what happens as we increase the number of dice. In the example with one die you should have seen a **Uniform Distribution** - each outcome was equally likely. What do you notice here as you increase the number of **samples** you are summing?

- Go from 2-31
- Is the distribution still uniform?
- Can you use the 68-95-99.7 rule to test what type of distribution you have? Can you come up with how to answer this question (write code)?

Uniformly Distributed data - more samples

```
The mean is: 3.973
The standard deviation is: 2.0050613456949393
```