Introduction to Mathematical Modeling

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Homework 4

Your task is to build a spreadsheet or python code that will calculate eigenvalues and solutions for the Annual Plants Model.

In class we formulated a model for annual plants with seeds that could germinate after either one or two years, this was the last model we considered on Day 5 (see class notes). This left us with a constant coefficient second order recurrence relation. We solved this using the eigenvalue method. Your goal is to experiment with the eigenvalues and solution to see what needs to happen for the plants to survive.

- First, go through the lecture notes and re-derive the equation that we found

$$\lambda = \frac{\alpha\sigma\gamma \pm \sqrt{(\alpha\sigma\gamma)^2 + 4\beta\sigma^2(1-\alpha)\gamma}}{2}$$

  See page 29 in the book scan for more help on this.

- Next build a spreadsheet or write python code that will automatically calculate the eigenvalues from the equation you just found. First input your parameters $\alpha = 0.5$, $\sigma = 0.8$, $\gamma = 2$, and $\beta = 0.25$ at the top of the spreadsheet.

- Assume that $P(0) = A$ and $P(1) = B$ are your initial conditions. Plug these initial conditions into the general form of the solution:

$$P(n) = C_1\lambda_1^n + C_2\lambda_2^n$$

  this should give you two equations for two unknowns. Show that you can solve these equations for the constants. Here is my form of the solution:

$$C_1 = A - C_2$$
$$C_2 = \frac{\lambda_1 A - B}{\lambda_1 - \lambda_2}$$

  Your job is to show how I got this.

- Now program your spreadsheet/code to automatically calculate your constants for the parameters that you choose at the top of the spreadsheet. (At this point you should have two eigenvalues and two constants being solved by your spreadsheet.)

- Now you have everything you need to calculate the population at any time step. Program the closed form solution into your spreadsheet.

$$P(n) = C_1\lambda_1^n + C_2\lambda_2^n$$

  You will have one column that is the year number, n, and another column that uses the equation above to calculate the population at step n. Find the population for the next 20 years. NOTE: Even if you can't get the previous parts you can plug into the equations I found and do the rest. (Please get help if you are having trouble!)

- Graph your solution using an xy-scatter plot.

- Finally, experiment with your parameters. How can you make sure that the population of plants does not die out? Does it mater what the initial population is? What parameters are most important in saving the population? What is your dominant eigenvalue and what does this value tell you? Type up your results. In the document include your graph and a discussion of each of the questions raised in this assignment.

NOTE: If you are having trouble you can read section 1.4.4 in the book, pages 25-30. This covers the model that we did in class and discusses the solution for one set of parameter values. You can use their parameter values to check that your spreadsheet is correct: same parameters = same results.

HELP: If you need help getting the spreadsheet started, I posted an example spreadsheet that calculates the eigenvalues and constants for the simpler case of $P(n+1) = aP(n) + bP(n-1)$. You can use this as a guide.

See next page for what to hand in!

PLEASE HAND IN - On Canvas:

1. A picture of your hand written calculations for the solution of the eigenvalues (following class notes) and the solution of the constants. Please show all your work and explain the calculations.
2. A graph of your results that shows the population over time for two interesting sets of parameters. For example, one set of parameters where the population survives and another where the population dies. This could just be a screen shot of your spreadshet
3. A written description of what you see in your graphs.
4. Your code. Either submit your spreadsheet as a .xlsx or send me a link to a cloud spreadsheet or google colab page with your work in it.