Introduction to Mathematical Modeling

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Project 3 Introduction

It is believed that Neanderthals were the original inhabitants of Europe and their species was very stable for more than 60,000 years. Forty thousand years ago the Neanderthals were replaced by our ancestors, the early humans (Cro-Magnon), who came to Europe from Africa much later. There is evidence that the two species coexisted in some parts of Europe. However, the mass extinction of the Neanderthals was rapid, in 5,000 to 10,000 years.

There are multiple theories for the demise of the Neanderthals:

1. Our first theory is that the Neanderthals and humans coexisted peacefully but because they were in competition for limited resources and early humans had better survival skills the Neanderthals became extinct. Basically nature and evolution took it's course.

2. Our second theory is that a genocide by the early humans that wiped out the Neanderthal population. Meaning that the early humans actively hunted the Neanderthals.

We are going to use mathematical modeling to investigate whether or not these theories are possible under basic assumptions. We begin by investigating the first theory:

Project 2 Specific Goals

We will assume that the Neanderthals, $N(n)$, and early humans, $H(n)$, lived in a limited resource environment and that the carrying capacity for the total humanoid population, $N(n) + H(n)$ was some constant value, K. Here n is measured in years. We will also assume that the total population satisfies the logistic equation, with the maximum growth rate, R. In each population we will allow for some natural deaths to occur with the mortality rate for the Neanderthals given by, M_1, and the mortality rate for the early humans given by, M_2. We can think of M_1 and M_2 as relating to survival skills. A lower mortality rate means that the group is better at survival. We will also assume, initially, that there is no hunting between the two groups.

The following system of equations describes the Neanderthal-early human interactions:

$$N(n+1) = N(n) + R(1 - (N(n) + H(n))/K)N(n) - M_1 N(n) \tag{1}$$

$$H(n+1) = H(n) + R(1 - (N(n) + H(n))/K)H(n) - M_2 H(n) \tag{2}$$

1. First convince yourself that these equations make sense. Why do we need the $N(n)+H(n)$ term as part of the logistic growth rate? What other assumptions are we making? Is this model linear or nonlinear? Make sure you can thoroughly describe the model, these equations, and all assumptions in your write up. You should also be able to draw a compartmental diagram.

2. Next, show that this system has the following fixed points:

$$\bar{N} = 0, \quad \bar{H} = 0$$
$$\bar{N} = 0, \quad \bar{H} = \frac{K}{R}(R - M_2)$$
$$\bar{N} = \frac{K}{R}(R - M_1), \quad \bar{H} = 0$$

In your write up discuss what each of these fixed points means in terms of the two populations. Which one of these fixed points models the case of the Neanderthals living without early human contact? You can use this value as your initial Neanderthal population, $N(0)$. NOTE: Calculating stability of fixed points is more complicated for systems of equations, but it is doable with a bit more math. For this project I am okay with you "testing" the stability using your spreadsheet simulations.

Also show, that a forth fixed "condition" is when $M_1 = M_2 = M$. In this case the populations eventually level out, but initially they grow or decay so that the total population is $\bar{N} + \bar{H} = \frac{K}{R}(R - M)$. Also, because they both grow and decay at the same rates, the ratio of $\frac{N(n)}{P(n)}$ remains the same as the initial ratio. What does this say about the possibility of coexistence?

3. Now write a spreadsheet to numerically calculate the solution to the model. Start by assuming that initially $H(0) = 10$ humans traveled to Europe and that the following parameters applied: $R = 1.2$, $K = 1000$. Initially we will also assume that both species have equal mortality rates, $M_1 = M_2 = 0.01$. Make sure to run your model out to at least $n = 5000$ since the extinction took between 5,000 and 10,000 years. What happens to the two species if they have the same mortality rates?

4. Next use your spreadsheet to determine if it is possible that slightly lower mortality rates for the early humans means extinction for the Neanderthals. For example let $M_2 = .009$ meaning a 10% decrease in mortality. How sensitive is your model to changes in the mortality rate? How sensitive is your model to the other parameters, such as growth rate or initial condition? Can you say anything about the stability of the fixed points? Does the first theory seem to be supported by your model? Make sure to report on a range of values and to experiment with your model.

5. Finally, lets investigate the idea of the early humans hunting the Neanderthals. Assume that each time they run into each other the early humans have a 0.0001 chance of killing the Neanderthal, and that the number of Neanderthals killed does not effect the growth of the human population. **You will need to make changes to the equations given above. Also convince yourself that this does not change your fixed points** Assume that the mortality rates are the same, $M_1 = M_2 = 0.01$. Find the fixed points of the new system. Do the fixed points change? In your spreadsheet, add the hunting term. What happens to the Neanderthal population? Does it take a long time for changes in the population to happen? Investigate how sensitive your model is to this hunting. Does the second theory seem to be supported by your model?

Write Up:

In your write up try to summarize what each model says about the theories **specifically noting the time that it takes for extinction to occur**. Make sure to discuss sensitivity of parameters and be aware of the time it takes for the populations to level out or go extinct under the different parameter values. Try to answer all the questions raised above. Usually in your **formulation** you will write down equations for the most general case and discuss all the parameters. Then begin the **results section** with a discussion of fixed points, followed by results and graphs for the populations over time from your spreadsheet. See if you can come up with interesting ways to represent the data. For example, you could plot the time for Neanderthal extinction as a function of early human mortality rate. You could go above and beyond by adding to the model. For example, another population, maybe a population of food source, or environmental factors that you could analyze.