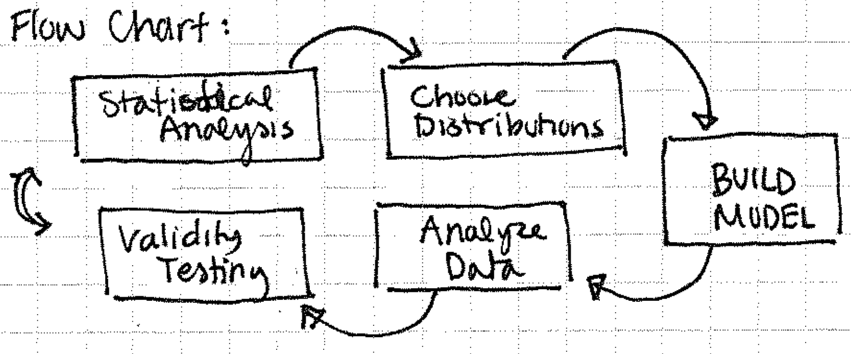
Take Quiz 1 → 11:00 – 11:20 ish. χ^2 table !!
when done please sit quietly or hang out outside.

Lecture 14 Mathematical Models

- Hand-in Outline/Assumptions
- Project 4 Due in 1 week
  - LOOK @ THE HINTS FOR HELP!

Flow Chart:



LAST TIME: BUILD THE MODEL.
Use the in class example spreadsheet to help in Project 4 ... similar commands.

TODAY. Validity Testing. P. 82 – 90 in the book.

- GENERAL IDEA → You build a model and want to know if you should trust it.
  So you run your model for a time period or data that you have.
  Check to see how well your model predicts the actual data.

- FIRST → What is the purpose of the model?
  - If you are just trying to show that a population will tend to die out ... validity testing is just double checking your assumptions. — THE DATA PATERN IS IMPORTANT.
  - If you are trying to predict a population # then you can do more accurate validity testing. — THE NUMBERS ARE IMPORTANT.

- OUR DATA FROM OUR MODEL DOESN'T MATCH THE REAL DATA EXACTLY – whether we are doing deterministic or stochastic.
  It never will !!!

How close is close enough?

one way to answer this question ... HYPOTHESIS TESTING

CHI - SQUARED (χ^2) - GOODNESS OF FIT TEST

$$\chi^2 = \sum_{i=1}^{n} \frac{(\text{Observed Value}_i - \text{Expected Value}_i)^2}{\text{Expected Value}_i}$$

n= number of data points
observed value — what the real data gives
expected value — what the model data gives.

REAL WORLD vs. MODEL WORLD.

THEN we look up our χ^2 value in the table:
$n-1$ = degrees of freedom
AT the top of the table α = significance level
confidence = $1-\alpha$.

HOW DOES THIS WORK...

1. Assume a HYPOTHESIS and write down the NULL HYPOTHESIS

- Scientific Method ... assume the worst

When testing we are asking. Is our model BAD

TAKE ACTION ← HYPOTHESIS: The model is a poor predictor of actual outcomes.

Something different or strange is happening

NULL HYPOTHESIS: The model still works and we will accept it untill we can find a reason to reject it.

DO NOTHING
The data fit theoretical expectations within some error.

Never accept the model as complete truth.

2. GATHER DATA — get real life data for the system AND run your model for the same data range or time period.

Model Data (EXPECTED)	Real. Data (OBSERVED)

3. Compute χ^2 for the data
   and determine the degrees of freedom
   n = # of data points
   $n-1$ = degrees of freedom

4. Look up α = significance level on the table

5. REJECT THE NULL HYPOTHESIS
   with an α significance level

AKA Our HYPOTHESIS is accepted at
a $1-\alpha$ confidence level.
AKA Our MODEL IS REJECTED with
a $1-\alpha$ confidence level.

ONE THING ⟶ you need to decide what
α is good enough for you... usually
Depends on the application 90% confidence
modeling squirrels vs heart valves!

EX The Cheater – a casino game involves rolling 3 dice.
For every six you roll you get \$10.
A casino tracks its gamblers to check for
cheating and they gather the
following data:

Real Life
vs. Model...
what we expect.

After playing 100 times

ONE PLAYER

48 rolls	0 sixes
35 rolls	1 six
15 rolls	2 sixes
3 rolls	3 sixes

This is the
real life data.

If we built a model for this game we could
use the binomial distribution to find the
probabilities.

$$f(k) = \binom{n}{k} p^k q^{n-k} \qquad \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$n=3$ three rolls of the dice each game
$p = 1/6$ probability of getting a six
$q = 5/6$ probability of faliure.

probability of zero sixes : $f(0) = 0.58$
one six : $f(1) = 0.345$
two sixes : $f(2) = 0.07$
three sixes : $f(3) = 0.005$

~~[illegible]~~ Just Plug into $f(k)$

so in 100 plays we would expect from our model.

58 rolls	0 sixes
34.5 rolls	1 six
7 rolls	2 sixes
.5 rolls	3 sixes

Our Hypothesis: The gambler was cheating — we have to do something. the model is a poor predictor of his outcomes.

Null Hypothesis : Our model is ~~correct~~ matches play and the gambler was not cheating. — do nothing. the model did predict his outcomes.

$$\chi^2 = \frac{(48-58)^2}{58} + \frac{(35-34.5)^2}{34.5} + \frac{(15-7)^2}{7} + \frac{(3-0.05)^2}{0.05}$$

$$\chi^2 = 23.367$$

$n=4$ data points $\Rightarrow$ 3 degrees of freedom

Look on the chart

0.05	0.025	0.01	0.005	α
7.815	9.348	11.345	12.838	χ^2

Better than this.

So we reject the null hypothesis with a better than 0.005 significance. → the model matched the real world with only .005 significance

<u>OR</u>

$(1-\alpha) = .995$

Have a better than 99.5% confidence in our HYPOTHESIS — the gambler was cheating.

99.5% confidence that the model is not predicting what we see.

We can use this type of testing on both deterministic and stochastic models.

We can also use this type of testing to choose between models.

Ex. Population Decline for a group of animals – we don't know why.

Stop here if no time.

Population starts at $P_0 = 100$

TWO THEORIES:

#1. The population has a constant decay rate – cut in ½ each year

$$P(n+1) = 0.5\,P(n) \longrightarrow P(n) = P_0(.5)^n$$

* HYPOTHESIS
  the model is wrong and we must choose a better one

* NULL HYPOTHESIS
  the model is acceptable for now and we do nothing.

#2. There is a linear trend to the population decline.

$$P(n) = 100 - \left(\frac{100}{7}\right)n$$

Year	Actual Data	Model 1	Model 2
0	100	100	100
1	53	50	85.71
2	27	25	71.43
3	15	12.5	57.14
4	9	6.25	42.86
5	3	3.125	28.57
6	1	1.5625	14.28
7	0	0.78125	0
8	0	0.390625	0

Which model is better?

NOTE... we need the expected value to be at least 5. Group data if needed.

We can calculate χ^2 for each model.

Model 1: GROUP DATA FOR YEARS 5–~~8~~

actual	expected
53	50
27	25
15	12.5
9	6.25
4	6.25

$\Rightarrow \chi^2 = 2.86$, $n = 5$ $\Rightarrow$ cut off for $\alpha = 0.25$ is $\chi^2 = 5.39$

The model matches the real world with .25 significance.

So we reject the null hypothesis with better than 0.25 significance

<u>or</u> we accept the hypothesis with 75% confidence

"not very confident"
"I am 75% sure that we should reject the model"

<u>or</u> Model 1 would be rejected less than 75% of the time.

- we fail to reject the model.
- <u>we accept our model with $\alpha = 0.25$ significance</u>

Model 2. No need to group data
all points $\geq$ 5.

actual	expected
53	85.71
27	71.43
15	57.14
9	42.86
3	28.57
1	14.28

$\Rightarrow \chi^2 = 133.2$, $n = 6$ $\Rightarrow$ cut off for $\alpha = 0.005$ is 18.548, $\alpha = 0.001$ in 22.46

So we reject the null hypothesis with less than 0.001 significance.

The model matches the real world with .001 significance

<u>or</u> we accept the hypothesis with 99.9% confidence

99.9% sure that the model is rejected.

<u>or</u> Model 2 would be rejected more than 99.9% of the time

we <u>successfully reject</u> the model.

- If we accepted this model it would be with $\alpha = 0.001$ significance.

NOTE: We can assume that 90% or $\alpha = .1$ is our cutoff

Often in more precise applications we would increase this or expect 95% or 99%

MODEL #1 matches our data better!