Lecture [redacted] 17 Math Models

~~First – Work on Matlab part of HW 9. due Tuesday~~ ~~Next.~~

- ~~Project 4 due Thursday 5pm.~~
- ~~Math modeling lab 5–8 pm~~

Today we start: Stages, States and Classes

- Until Now... all of ou models had the entire population growing the same way.
- In reality... different parts of the population grow differently.

EXAMPLES.

Investments: you don't usually invest all of your money into a single account... different accounts earn different amounts of interest.

Populations: different members of the population reproduce differently. some are more suseptable to disease

Medicine: different body tissues react differently to drugs... absorption, damage etc...

How do we build one of these models?

<u>Example</u> Human Population – Female age-class model

- many population models only consider the number of females when determining population ... the # of births determined by the # of Females
- the probability of a female giving birth or dying changes over the course of her lifetime.

#1 Choose the # of age classes you want ...

Insurance companies have birth and death rates for people up to age 102 !

We will start simple and consider only three age classes...

Class size determines time step.

CLASS 1 0 – 15 years
CLASS 2 15 – 30 years
CLASS 3 30 – 45 years

want the groups to all be the same size ! (15 years)

OR we must account for this in the model

- Assume child bearing stops @ age 45
- We are tracking the population of women who could have a child.
- Because our groups are 15 years wide we step in time by 15 years.

From: "Introduction to the mathematics of Demographics" – RL Brown.

Variable Name	Age Group	Birth Rate	Survival Rate
P_1	0 – 15	0.4271	0.9924
P_2	15 – 30	0.8498	0.9826
P_3	30 – 45	0.1273	——

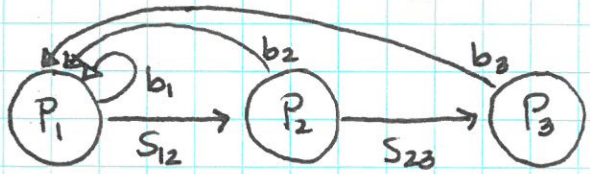
the 15-year time steps are built into these #'s.

How does the population change?

Q. How does one become a member of P_1 ? – by being born to a mother in any group.

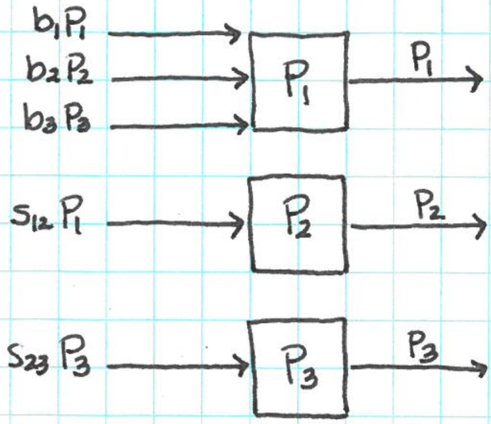
Q. How does one become a member of P_2 ? – by surviving P_1.

Lets draw a picture... $\boxed{\text{STATE DIAGRAM}}$ shows how a system moves from state to state.



Pictures are very helpful in creating a model!!

VS. COMPARTMENTAL DIAGRAM



Note: at each time step... all of the population moves to the next age group... no lying about your age!

NOTE
Here our stages are same length in the paper the have varried lengths.

RECURRENCE RELATIONS

$$P_1(n+1) = b_1 P_1(n) + b_2 P_2(n) + b_3 P_3(n)$$

$$P_2(n+1) = S_{12}\, P_1(n)$$

$$P_3(n+1) = S_{23}\, P_2(n)$$

Assume $P_1(0) = P_2(0) = P_3(0) = 30$

we can still model this using excell.

EXCELL SPREAD SHEET EXAMPLE...

1. You need one collumn for each state.
2. Your parameters are now a table of valves at the top of your spreadsheet
3. Often we are interested in the long term population percentages.

NOTE... changing the initial population does not ~~change~~ change the ~~lo~~ long term %.

An important tool in stage Models: MATRICEES !!

I can write my recurrence relation in matrix form:

NOTE: Must have a linear model.

1. Re-write the recurrence relations and LINE UP the state variables.

$$P_1(n+1) = b_1 P_1(n) + b_2 P_2(n) + b_3 P_3(n)$$
$$P_2(n+1) = S_{12} P_1(n) + \emptyset P_2(n) + \emptyset P_3(n)$$
$$P_3(n+1) = \emptyset P_1(n) + S_{23} P_2(n) + \emptyset P_3(n)$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

2. Read the matrix from this ... RHS matrix entries are just the parameters!

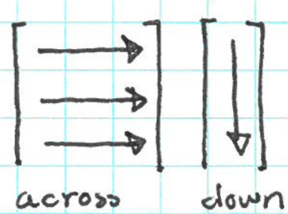
$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}_{n+1} = \begin{bmatrix} b_1 & b_2 & b_3 \\ S_{12} & 0 & 0 \\ 0 & S_{23} & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

"Population Projection Matrix"

$\vec{P}_{n+1} = A \vec{P}_n$ this is our shorthand form!

Matrix multiplication for each time step! This idea is AWESOME for computers.

REVIEW OF MATRIX MULT $\rightarrow$ Visually:



mult components and add together

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times2 + 2\times1 \\ 3\times2 + 4\times1 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} b_1 & b_2 & b_3 \\ S_{12} & 0 & 0 \\ 0 & S_{23} & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} b_1 P_1 + b_2 P_1 + b_3 P_3 \\ S_{12} P_1 + 0 + 0 \\ 0 + S_{23} P_2 + 0 \end{bmatrix}$$

we get our recurrence relations back when we do matrix mult w/ parameters

MATLAB EXAMPLE.

next time eigenvalues/vectors.

MATLAB MODEL


```
% This is a Matlab script to model a deterministic population
% with logistic growth and harvesting


% DEFINE PARAMETERS

R=1;   % maximum growth
K=100;  % carying capacity
d=.3;   % percentage of harvesting

P(1)=50; % initial population

EndTime=50; % number of years to model

% Now step through time

for n=1:EndTime

    % Enter the recurrence relation exactly

    P(n+1)=P(n) + R*(1-P(n)/K)*P(n) - d*P(n);

end

% Graph the final result

plot(P,'*-')
xlabel('Time')
ylabel('Population')
axis([0,EndTime,0,K])
```

Computer Reads Line by Line

Double check capital letters and misspellings.

```
function [] = untitled()

NUMSTAGE=3; % This is the number of stages that your population has.
NUMSTEPS=20; % This is how many time steps you want to do
% Initial Population for Each Stage
P1=30;
P2=30;
P3=30;
% If I had more stages I would add another initial population for each
% stage.
%P4=
%P5=
%P6=

% Build the population matrix...
A=zeros(NUMSTAGE,NUMSTAGE);

% Put numbers into the population matrix row by row...
A(1,:)=[0.4271, 0.8498, 0.1273];
A(2,:)=[0.9924, 0,      0     ];
A(3,:)=[0,      0.9826, 0     ];
% IF I had more stages I would add another row for each stage
% A(4,:)=[]
% A(5,:)=[]
% A(6,:)=[]

% Fill in data for the initial population
P(1,1)=P1;
P(1,2)=P2;
P(1,3)=P3;
% If I had more stages I would need more initial population inputs
%P(1,4)=P4;
%P(1,5)=P5;
%P(1,6)=P6;


% A matrix to track time
T(1)=0;

for i=1:NUMSTEPS
    % Each time step we just do a matrix multiplication.
    P(i+1,:)=A*transpose(P(i,:));
    % here we must transpose X so that the matrix multiplication works out!

    % Move our time one step forward
    T(i+1)=T(i)+1;

end


% Print the new matrix
% Anything with out a semicolon at the end will print!
FinalPOP=P(NUMSTEPS+1,:)

% Plot the Population for each stage
for j=1:NUMSTAGE
```

```
    plot(T(:),P(:,j), 'ko-','LineWidth',2);
    axis=[0, max(max(P)), 0, max(T)];
    hold on
end



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%EIGENVALUES - this will automatically calculate and normalize your
%eigenvalues - the largest eigenvalue is associated with the long term
%population percentages!

[V,E]=eig(A);

% Normalize vectors
for j=1:3
    NormV(:,j)=V(:,j)./sum(V(:,j));
end

DOMINANTEigenValues=E(1,1)

DOMINANTEigenVectors=NormV(:,1)



%Prompt user to save the new matrix
checkLabels = {'Save Popluation matrix to variable named:'};
varNames = {'POP'};
items = {P};
export2wsdlg(checkLabels,varNames,items,...
             'Save to Workspace');

%Prompt user to save the new matrix
checkLabels = {'Save Eigenvalues to variable named:'};
varNames = {'EIG'};
items = {E};
export2wsdlg(checkLabels,varNames,items,...
             'Save to Workspace');

%Prompt user to save the new matrix
checkLabels = {'Save Normalized Eigenvectors to variable named:'};
varNames = {'EVEC'};
items = {NormV};
export2wsdlg(checkLabels,varNames,items,...
             'Save to Workspace');

end
```

Lecture ~~18~~. 67?

Build a Stella Model
- Together Human Pop... Sheet1

- Hand out Project 5
- Sign up sheet Meeting Times

	P1	P2	P3
Initial Population	30.0000	30.0000	30.0000
Birth Rate	0.4271	0.8489	0.1273
Survival Rate	0.9924	0.9826	----------

	0-15	15-30	30-45	TOTAL
0	30.00	30.00	30.00	90.00
15	42.10	29.77	29.48	101.35
30	47.01	41.78	29.25	118.04
45	59.27	46.65	41.05	146.97
60	70.14	58.82	45.84	174.79
75	85.72	69.61	57.79	213.12
90	103.06	85.07	68.40	256.52
105	124.94	102.27	83.59	310.80
120	150.82	123.99	100.49	375.30
135	182.46	149.68	121.83	453.97
150	220.50	181.08	147.07	548.65

Percentage of total	0.4019	0.3300	0.2681