Lecture 19 Math Models

- Project 5 Due Tuesday November 23rd 5pm
- Final Project Meetings (sign up)
  - Have Data
  - Draw diagrams
  - Write down equations.
  - Show me your write up about Loggerhead Turtles.

LAST TIME: Introduced STELLA ... remember Applications/Stella9/Tutorials/Model Building Tutorials.

lots of help here.

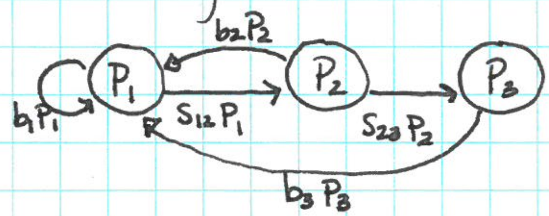
You may use what ever program you want for Project 5 → make sure your group agrees and EVERYONE knows how to build the model.

Today: A few examples of more complicated examples.

<u>EX</u> How do we model male and female populations ?

- Female only model :

STATE DIAGRAM

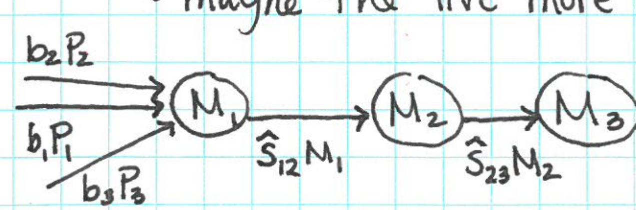


- IF we assume that males and females exist in equal #'s and have the same survival rates then

$M_1 = P_1 \quad M_2 = P_2 \quad M_3 = P_3$ no aditional calculation needed!

- What if the males have different survival rates?
  - maybe the live more dangerous lives?

$b_1 = \frac{\text{true. birthrate}}{2}$ already built into models.



Assume 50/50 chance of being born male or female.

Recurrence Relations:

$P_1(n+1) = b_1 P_1(n) + b_2 P_2(n) + b_3 P_3(n)$ # of females born...
$P_2(n+1) = s_{12} P_1(n)$
$P_3(n+1) = s_{23} P_2(n)$

$M_1(n+1) = b_1 P_1(n) + b_2 P_2(n) + b_3 P_3(n)$ # of males born is the same as # of females
$M_2(n+1) = \hat{s}_{12} M_1(n)$
$M_3(n+1) = \hat{s}_{23} M_2(n)$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix}_{n+1} = \begin{bmatrix} b_1 & b_2 & b_3 & 0 & 0 & 0 \\ s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & s_{23} & 0 & 0 & 0 & 0 \\ b_1 & b_2 & b_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{s}_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & \hat{s}_{23} & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix}_n$$ Matrix Form.

NOTE... Draw the picture first! MOST IMPORTANT STEP
The write down equations
an if needed write in matrix form.

<u>EX</u> How do we model populations that can skip a state?

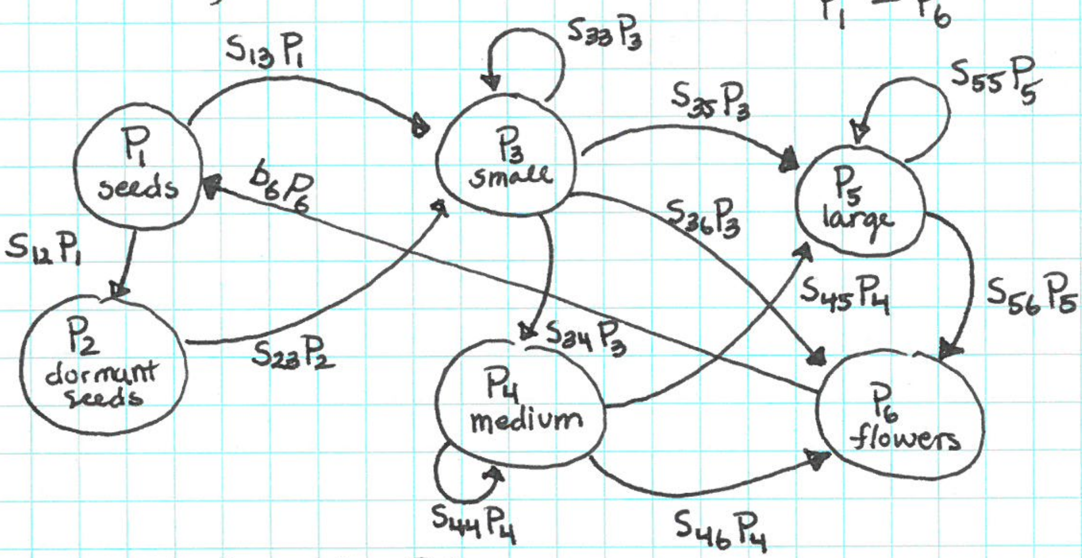
A species of perennial plant has 6 possible stages:
- First-year dormant seeds
- Second-year dormant seeds
- Small non-flowering rosettes
- Medium non-flowering rosettes
- Large non-flowering rosettes
- Flowering Plants

- Not all seeds are dormant for two years some become small rosettes after just one year.
- The plants can skip some of the non-flowering states ... they all start small but some can skip to flowering while some will remain in one of the non-flowering states

- Only flowering plants produce seeds, die after one year
- plants can remain in a rosette state for multiple years.

Together we will draw a state diagram...

How many states do we have? Six, call them $P_1 - P_6$



How do we get to being in P_1?
- must be a seed produced by P_6!

What about P_2?
- we survived P_1 <u>and</u> remain dormant.

What about P_3?
- either we survived P_1 <u>and</u> became a small <u>OR</u> we survived P_2 <u>OR</u> we stayed in P_3

What about P_4?
- we came from P_3 <u>OR</u> we stayed in P_4

What about P_5?
- we came from P_4 <u>OR</u> we stayed in P_5 <u>OR</u> we skipped P_4 and came from P_3

What about P_6?
- we came from P_5 <u>OR</u> we skiped P_5 and came from P_4 <u>OR</u> we skipped both $P_4 + P_5$ and came from P_3.

What do the recurrence relations look like?

STATE DIAGRAM ... only the inflow matters!

$$P_1(n+1) = b_6 P_6$$
$$P_2(n+1) = S_{12} P_1$$
$$P_3(n+1) = S_{13} P_1 + S_{23} P_2 + S_{33} P_3$$
$$P_4(n+1) = S_{34} P_3 + S_{44} P_4$$
$$P_5(n+1) = S_{35} P_3 + S_{45} P_4 + S_{55} P_5$$
$$P_6(n+1) = S_{36} P_3 + S_{46} P_4 + S_{56} P_5$$

Once you have the picture you can write down equations easily.

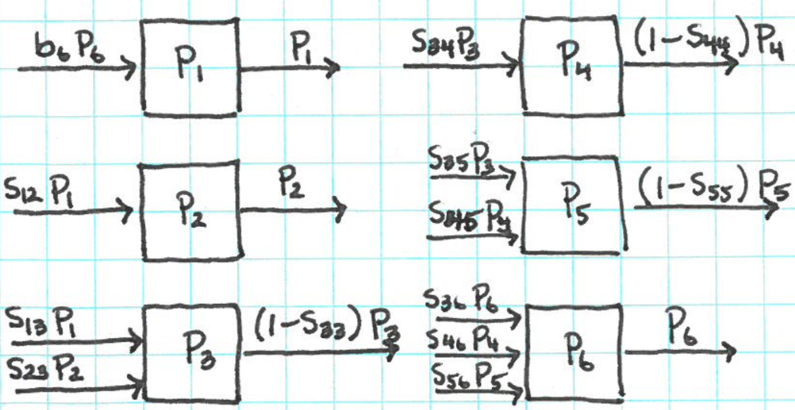
[Excell spreadsheet]

Write this in Matrix Form

$$\vec{P}_{n+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & b_6 \\ S_{12} & 0 & 0 & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & S_{34} & S_{44} & 0 & 0 \\ 0 & 0 & S_{35} & S_{45} & S_{55} & 0 \\ 0 & 0 & S_{36} & S_{46} & S_{56} & 0 \end{bmatrix} \vec{P}_n$$

[Matlab/Octave, Java, Python...]

How do we draw compartmental diagrams?



[Stella Model]

$b_6 = 322.38$
$S_{12} = 0.966$
$S_{13} = 0.013$
$S_{23} = 0.010$
$S_{33} = 0.125$

$S_{34} = 0.125$
$S_{44} = 0.238$
$S_{35} = 0.245$
$S_{45} = 0.167$
$S_{55} = 0.862$

Program this on Stella

$S_{36} = 0.125$
$S_{46} = 0.245$
$S_{56} = 0.278$