Math Modeling Lecture 2.

HW Read 1.1
Work on eqns for ping pong model.

• Take a few minutes to discuss the homework...

What assumptions did you make?
What resources did you have?
What were your answers?

Full credit for attempting each problem.

Some Important Ideas from the reading:

1. Every model is a PARTIAL REPRESENTATION of the world around us.

2. RESOURCE CONSTRAINTS are a big issue that every modeler has to deal with.
   • I expect much more thought and analysis in the 1 week ping pong model.

3. Every model has ASSUMPTIONS, they are essential but they must make sense.

OCCAMS RAZOR – William of Occam had a "sharp and cutting mind"

"Entities should not be multiplied beyond what is necessary"

Application to Science → The more simple explanation is probably the correct one.

Application to Modeling → Elliminate unnecessary info

<u>BAD MODEL</u> – cuts out too much or too little

<u>GOOD MODEL</u> – strikes a good ballance.

• keep in mind your PURPOSE AND CONSTRAINTS when making assumptions ← not just lazy assumptions.

Important types of Models:

Static vs. Dynamic — A <u>static model</u> does NOT incorporate time as an independent variable.
Ex our ping pong model
A <u>dynamic model</u> is time dependent
Ex modeling growth over time.

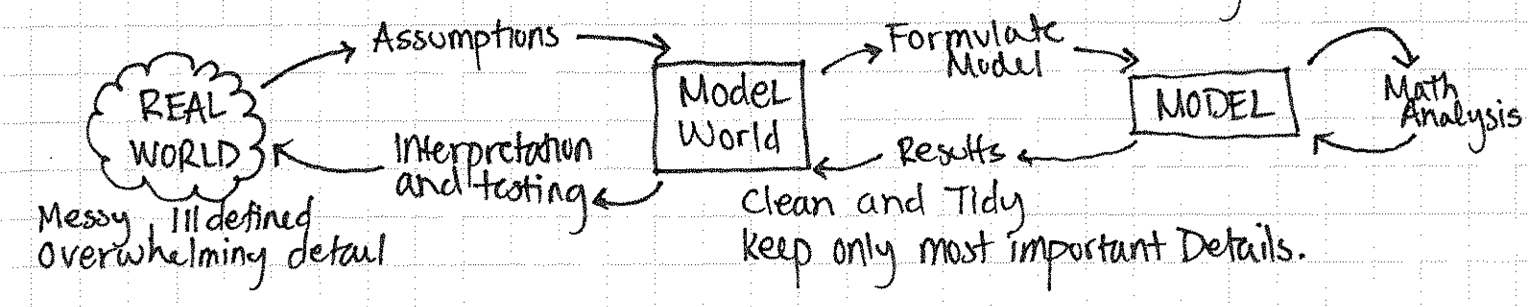
Continuous vs Discrete — this has to do with how we measure time in a dynamic model
<u>continuous</u> — time is measured continuously like we experience it in life ... uses calculus and differential eqns.
<u>discrete</u> — time is measured in integer steps like we would experience if we walked around blinking our eyes

deterministic vs Stochastic — A <u>deterministic</u> model has no randomness we use average values to represent the whole population
A <u>stochastic</u> model allows for randomness either by allowing for catastrophies or assuming that the population is represented by a probability distribution.

Our ping pong model is a Static Deterministic Model.

Next time we will start talking about Dynamic, Discrete Deterministic models.

(ON THE SIDE) A more specific overview of the modeling process:



ALWAYS START SIMPLE then ADD IN DETAIL

<u>NOTE</u> — do 1 min and 5 min brainstorm each time. this really helps.

The Importance of FORMULAS and DIAGRAMS.

- each mathematical model should include both a formula and diagram.

Ex Draw a diagram of the room along with measurements.

- the idea is that you will come up with a formula that can be applied to any system

Ex
$$\#\text{ of ping pongs} = \frac{(\text{Total volume of room}) - (\text{Volume used up by desks})}{(\text{Volume of a square ping pong})}$$

Your's should be more complicated than this!

$$P = \frac{V_R - V_D}{V_P}$$

this is a formula that you can apply anywhere and analyze!

Ex An example of the full modeling process...

THE QUESTION: How long does it take to go on a hike?

Take a minute and brainstorm:
What info do you need?
What might effect the hike?

STATIC DETERMINISTIC MODEL:

START SIMPLE: Most important: DISTANCE and AVERAGE SPEED

$$\text{TIME} = \underset{(\text{miles})}{\text{DISTANCE}} \times \underset{(\text{minutes/mile})}{\text{TIME TO WALK ONE MILE}}$$

- write it out in words

what assumptions are we making?
Then define varriables and parameters.

T = time D = distance w = walking speed.

OUR MODEL: $\boxed{T = Dw}$

This model applies to any hike and any person... we will plug in # later.

NOW REASSESS OUR ASSUMPTIONS...

a lot more happens on a hike than just walking... lets add in some more complexity.

Lets account for taking photos on the hike.

TIME SPENT TAKING PHOTOS = # of PHOTOS PER MILE (photos/mile) x TIME TO TAKE A PHOTO (~~minutes~~/photo) x DISTANCE (miles)

it helps to check units!!

P = # of photos per mile
s = time to take a photo

Time spent taking photos $= DPs$

OUR NEW MODEL:

$\boxed{T = Dw + DPs}$

- this still applies to any person / hike.

GREAT given a person, distance of hike, # of photos etc ... we can give a good estimate of the length of the hike.

EVEN BETTER because we have an equation we can ask more complicated questions!

MATH ANALYSIS ... USE THE MODEL

Ex What reduces hiking time more, doubling the hiking speed or taking half as many photos?

- once you build your model think of questions you can ask.

Ask questions of the ping pong model.

$w =$ walking speed $\left(\frac{\text{minutes}}{\text{mile}}\right)$ $\qquad \frac{w}{2}$ doubles walking speed

$p =$ # of photos $\left(\frac{\text{photos}}{\text{mile}}\right)$ $\qquad \frac{p}{2}$ reduces # of photos by half.

COMPARE THESE CASES:

$$T_1 = \text{Time w/ double speed} = D\frac{w}{2} + Dps$$

$$T_2 = \text{Time w/ } \tfrac{1}{2} \text{ photos} = Dw + Dp\frac{s}{2}$$

NOW COMPARE THESE CASES:

CASE 1 $\quad T_1 > T_2 \quad$ Better to ~~double the speed~~ reduce # of photos... if is the shorter hike.

$$D\frac{w}{2} + Dps > Dw + Dp\frac{s}{2}$$

plug in and simplify

$$\frac{w}{2} + ps > w + p\frac{s}{2}$$

$$w + 2ps > 2w + ps$$

$$ps > w \qquad \left(\frac{\text{photos}}{\text{mile}} \cdot \frac{\text{minutes}}{\text{photo}}\right) > \left(\frac{\text{minutes}}{\text{mile}}\right)$$

Time spent taking photos each mile > Time spent walking each mile

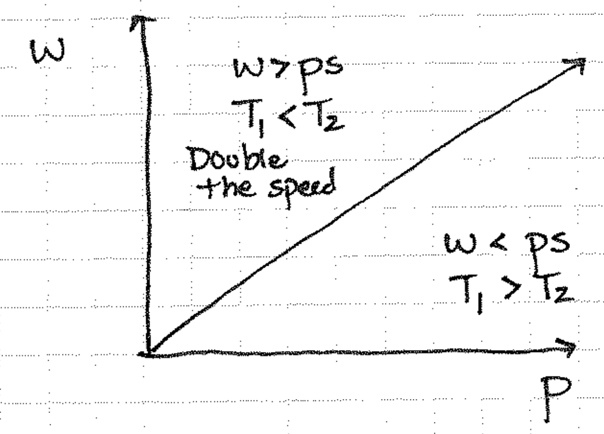
This makes sense: if the time spent taking photos each mile is greater that the time spent walking each mile then it reduces more hike time if you take ½ as many photos.

CASE 2 $\quad T_1 = T_2 \quad ps = w$

CASE 3 $\quad T_1 < T_2 \quad ps < w$

The algebra remains the same.

Lets see this on a graph ... we are discussing
w = walking speed and P = photos per mile
put these on the axis ...

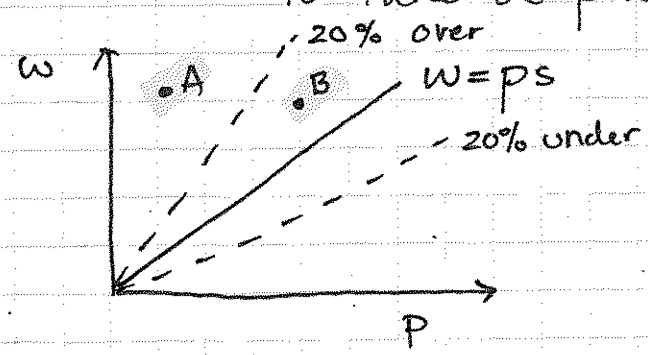


graph the equality.
$w = ps$ s = time to take a photo is a parameter
↓
determines slope of line.

now given a person we can see which option is best for decreasing hiking speed.

<u>also</u> – notice what doesn't appear: Distance
the length of the hike doesn't matter to our question.

FINALLY ADDRESS THE QUESTION — how good is my model?
How sensitive is the model to s – time to take a photo.
– lets assume we can know, s, the time to take a photo w/ 20% accuracy.



Knowing our error or at least addressing the idea by estimating it gives us a
REGION OF CERTAINTY

What does this mean?

<u>Ex</u> Consider friend A ... a very slow walker who doesn't take many photos
A is way outside the dotted line ... so we are sure that it is better to double their speed even with the possible error in estimating photo time!
In this region the model is ROBUST
small changes or errors don't change the soln.

Ex Consider friend B ... A slow walker who takes more photos.

B is within the dotted lines so if we were a little off in estimating time to take a photo our solution would change!

In this region the model is SENSITIVE small changes or errors do change the soln.

Sometimes just defining what variables are ROBUST and which are SENSITIVE is hugely important!