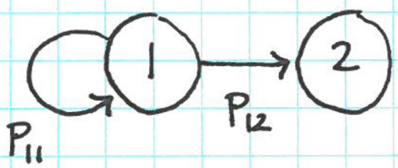
Check Moodle project prompts

Lecture 20 Math Models

Last Time:
More Complicated examples of stage based models.
Perenial Plant Model.

• Project 5 Due Nov 24th 5pm (I will be away mon + tues)

Today: Markov Models ... similar to stage models but we think of transitioning from one location to another



The animal can either remain at location 1 or move to location 2.

Not modeling the loss of animals: $P_{11} + P_{12} = 1$

• The sum of all arrows out of a state must equal one, the animals must go somewhere.

The solution vector in a markov model lists the probability that a system is in a certain state @ a certain time <u>OR</u> the percentage of the system in each state.

1. Markov assumption – the probability of moving from one state to another is independent from how you got to the first state.

$$\vec{X}_{n+1} = T\vec{X}_n$$

$\vec{X}$ = solution vector
T = transition matrix

• we still draw state diagrams and use recurrence relations / matrix

How do we talk about a "steady state" long term solution for a Markov Process?

Same as before...

1. Do the time stepping and see what happens ... assuming the solution tends to a steady state

$$\vec{x}(1) = T\vec{x}(0)$$
$$\vec{x}(2) = T\vec{x}(1) = T^2\vec{x}(0)$$
$$\vdots$$
$$\vec{x}(n) = T^n\vec{x}(0)$$

Long term soln $\bar{x} = L\bar{x}$

soln does not change from one step to the next.

This is like T^n as $n \to \infty$

$L \sim T^n$

choose large n

compute T^n.

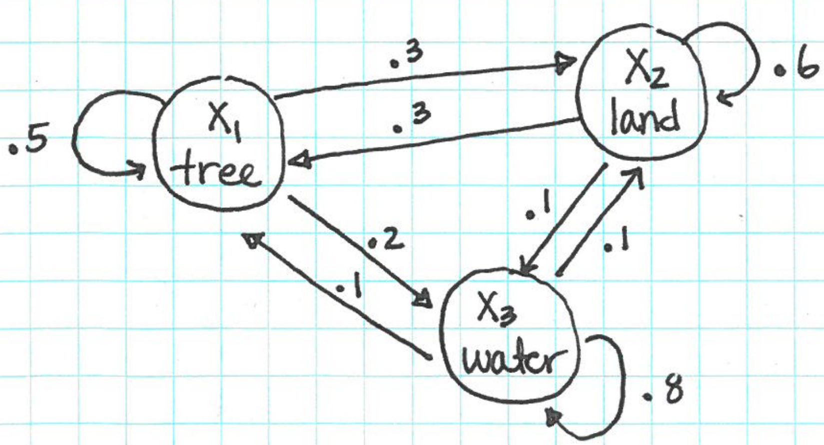
columns of this give %

2. Eigenvalues and Eigenvectors

For a markov process the dominant eigenvector is $\lambda = 1$

The associated eigenvector is the long term probabilities.

<u>Ex</u> An animal traveling through regions.



If the animal is in the tree:

- 50% probability of staying in the tree
- 30% probability of moving to land
- 20% probability of moving to water.

Recurrence Relations:

$$x_1(n+1) = .5\,x_1(n) + .3\,x_2(n) + .1\,x_3(n)$$
$$x_2(n+1) = .3\,x_1(n) + .6\,x_2(n) + .1\,x_3(n)$$
$$x_3(n+1) = .2\,x_1(n) + .1\,x_2(n) + .8\,x_3(n)$$

In Matrix Form:

$$\vec{x}_{n+1} = \underbrace{\begin{bmatrix} .5 & .3 & .1 \\ .3 & .6 & .1 \\ .2 & .1 & .8 \end{bmatrix}}_{T\text{ - transition matrix}} \vec{x}_n$$

ON MATLAB: eigenvalues $\lambda_1 = 0.2438$ $\lambda_2 = 1$ $\lambda_3 = 0.6562$

eigenvectors (normalized) $V_1 = \begin{bmatrix} X \end{bmatrix}$ $V_2 = \begin{bmatrix} 0.2692 \\ 0.3077 \\ 0.4231 \end{bmatrix}$ $V_3 = \begin{bmatrix} X \end{bmatrix}$

dominant evec - long term %'s.

even though the animals may be moving... they keep to this ballance.

26.92% of the population in tree
30.77% of the population on land
42.31% of the population in water

we could also compute T^n for n large
then $L \sim T^n$ ~~[illegible]~~ would be a good approximation to $\vec{v}_2$

Aside... what if I wanted an animal to take a "random walk"

STOCHASTIC

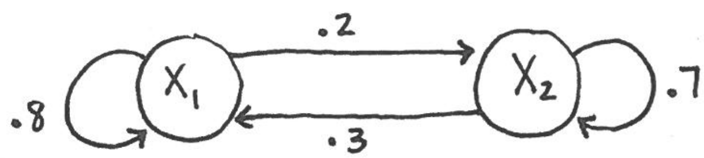
each step do $P = \text{rand}()$ $\longrightarrow$ if ($P \leq .5$, $P_{23} = 1$, $P_{23} = 0$)
if ($P \geq .5$, $P_{12} = 1$, $P_{12} = 0$)

we can imagine doing this in 2-D also.

Math 231
Matlab Example - Markov Models

Goal: Use Matlab to analyze the following Markov type models.

1.



(a). Write down the recurrence relations for the system drawn above:

$$x_1(n+1) =$$
$$x_2(n+1) =$$

(b). Write the system in matrix form:

$$\vec{x}_{n+1} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \vec{x}_n = T\vec{x}_n$$

(c). Using Matlab:

- Enter the matrix T:

$$T(1,:) = [\ \ ,\ \ ];$$
$$T(2,:) = [\ \ ,\ \ ];$$

- Find eigenvalues and eigenvectors:

$$[d, v] = eig(T) \tag{1}$$

- Normalize the dominant eigenvector (located in column a):

$$Norm = d(:,a)./sum(d(:,a)) \tag{2}$$

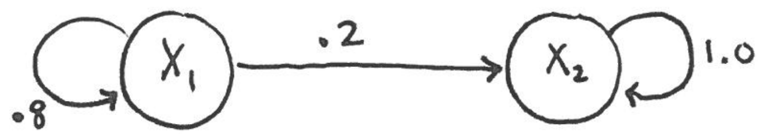
- What are the long term probabilities for this system?

(d) Find L such that ~~if~~ $\bar{x} = L\bar{x}$ where $L \sim T^{100}$. (In Matlab type: $L = T^{100}$.)

(e) Does L match what you found from your normalized eigenvector?

(f) On Blackboard under In Class Documents and In Class Examples, download the STELLA program for this system. (Markov_ example.STM) Run the program in STELLA. Does this match what you found in (c) and (d)?

2. Do the same analysis (parts (a)-(f)) for the following system: (Note: you will have to alter the STELLA program to do part (f).)



3. In the system from problem 2, we call x_2 as "absorbing state". Why?

4. For the following matrix, Markov system, are there any absorbing states? How do you know?

$$\vec{X}_{n+1} = \begin{bmatrix} .2 & 0 & .1 & 0 \\ .1 & 1 & 0 & 0 \\ .3 & 0 & .9 & 0 \\ .4 & 0 & 0 & 1 \end{bmatrix} \vec{X}_n \qquad \vec{X}_n = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

5. Draw a state diagram for the system in part 4.