Math Modeling Lecture 3

Go To: Science Poster Session 7-9 pm

Recall Last Time –

- STATIC
- DYNAMIC
- DETERMINISTIC
- STOCHASTIC
- DISCRETE
- CONTINUOUS

retrieve your memory what do each of these mean?

We also built a static, deterministic model of going hiking and taking photos:

$$T = Dw + Dps$$

D = distance
w = walking speed
p = # of photos
s = time to take photo

We analyzed this and discussed ROBUST vs. SENSITIVE

NOTE - once you build your model you test which parameters are sensitive and which are robust.

TODAY: DISCRETE DYNAMICAL SYSTEMS.

- a discrete model is best when time progresses slowly of progress is made in steps.

EX	SWIRLING OF WATER IN A CUP	GROWTH OF NAPLE TREES
How do we measure it?	Put a meter in at one point collect lots of data	Choose a plot of trees count the # of trees
How does it change	Constant change. The swirling slows down quickly.	In a day not much change. One real growth season.
What type of Model	Continuous	Discrete.

An important tool in discrete models:

RECURRANCE RELATIONS or DIFFERENCE EQNS.

- a function is recursively defined if $X(n)$ is a function of the value of X at some of the values less than n.

NOTATION: $X(n) = f(X(n-1), X(n-2), \ldots, X(1), X(0))$

$X_n = X(n)$ where n is an integer value that denotes the step #.

(we will use)

Ex Fibonacci Sequence: each term is the sum of the two previous terms.

$$X(n) = X(n-1) + X(n-2)$$

- to get the sequence going we need the first two numbers

INITIALIZATION (initial conditions) $X(0) = 1 \quad X(1) = 1$

Then $X(3) = X(2) + X(1) = 1 + 1 = 2$

$X(4) = X(3) + X(2) = 2 + 1 = 3$ and so on...

The ORDER of a recurrance relation is the # of previous time steps that the relation makes reference to.

$X(n) = X(n-1)X(n-2) - X(n-1)$ ORDER 2

$X(n) = X(n-1) + X(n-2) - X(n-3)$ ORDER 3

ORDER # = # of Initial Conditions.

Ex Factorial: $X(n) = n \cdot X(n-1)$

What is the order?

If $X(0) = 1$ calculate $X(5)$

$$X(5) = 5X(4) = 5 \cdot 4X(3) = 5 \cdot 4 \cdot 3 \cdot X(2) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot X(1)$$
$$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot X(0) = 120.$$

- All sorts of things can be modeled using discrete models and recurrence relations.

NEXT TIME: Discrete Population Models.

TODAY: → START LEARNING EXCEL ... LIBRE OFFICE.

BUDGET – projecting finances into the future... HOMEWORK 2.

- first we will start with some brain storming and build our model
- then we will use a spreadsheet program to project into the future.

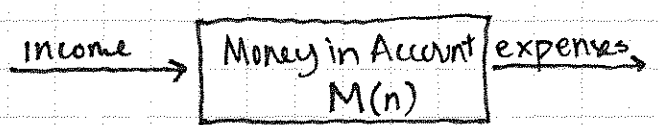
- What things are important in predicting how much money you have in the future?

INCOME EXPENSES INTEREST RATES

STARTING # LARGE PURCHASES LIFE CHANGES.

START SIMPLE – Income and Expenses

1. DRAW A COMPARTMENTAL DIAGRAM:



- Box contains the variable that you are tracking.
- In arrows – how the quantity increases
- Out arrows – how the quantity decreases

Define Variables.

M = # of dollars

n = month #

2. WRITE THE FORMULA IN WORDS → THEN VARRIABLES.

The Change in Money in my Account = Income − Expenses

$$M(n+1) - M(n) = I - E$$ Difference Equation

$$\boxed{M(n+1) = M(n) + I - E}$$ Recurrance Relation.

$$\boxed{M(0) = M_0}$$

3. NOW REFINE THE MODEL - we need a formula for Income and Expenses.

INCOME:

HOURLY WAGES: $p \times h$ — p = hourly pay (dollars), h = hours worked (per month)

SCHOLARSHIP: YEARLY: $S/12$ — S = scholarship amount. (per year)

HELP FROM PARENTS: H_p (dollars each month)

Later add ... INTEREST FROM SAVINGS: $r M_s(n)$ or ONE TIME INCOME etc ...

$$I = ph + S/12 + H_p$$

EXPENSES:

RENT: R
UTILITIES: u
FOOD: F
} dollars

TUITION: $T/6$ — T = one semester tuition.

FUN $: F_M

$$E = R + u + F + T/6 + F_M$$

Later add ... ONE TIME EXPENSES ... MONEY PUT INTO SAVINGS ... etc ...

4. PUT IT ALL TOGETHER

$$M(N+1) = M(N) + ph + S/12 + Hp - R - U - F - T/6 - F_M$$

$M(0) = M_0$ = how much $ I have now

5. PROGRAM A SPREADSHEET TO PROJECT INTO THE FUTURE.

- LIBRE OFFICE / OPEN OFFICE / EXCEL

A. open the spreadsheet and start by entering the parameter on the first sheet.
- These are things that you might change or experiment with.

B build your model on the next sheet.
- each cell can be programmed to look back at your first sheet
- use $ $ to make static references.
- The FIRST month always looks at the initial condition - ALL other moths look only at the previous month.

C. Go back to the first sheet to do your graphs and analysis.
- How much $ will I have in 1 year? when I finish school?
- How many hours do I have to work to break even?