Lecture 4 Math Modeling

Read 11-28
Take notes try
The examples.

[redacted] in

[redacted] tomorrow.

Hand in Project 1 & Do a Self Assessment.
Spreadsheet HW Due Thursday

LAST TIME: Discrete deterministic dynamic models.
What do these things mean?

- Projecting your Budget into the future.
- Started working with spreadsheet

HINTS:
- Always separate out your model and your parameters.
- After building the model USE IT to answer questions.

NOTE... most of our models are going to be done in the spreadsheet... but if you like to program choose the programming language you want.

DISCRETE DETERMINISTIC POPULATION MODELS

START SIMPLE What is our starting point?
How does population change?

Brainstorm:
P_0 = initial population
Births
Deaths
Catastrophes – Food Shortage
Climate Change
Migrations

$$\xrightarrow[B]{\text{Births}} \boxed{\begin{array}{c}\text{Population}\\ P(n)\end{array}} \xrightarrow[D]{\text{Deaths}}$$

n = number of years.

$$P(n+1) - P(n) = B - D \quad \text{difference Eqn.}$$

Most simple Model.
$$\begin{cases} P(n+1) = P(n) + B - D & \text{recurrence Relation.} \\ P(0) = P_0 \end{cases}$$

NOW REFINE: We need an equation for Births (B) and Deaths (D)

- Lots of ways to do this
  we can include lots of ideas.

What is the most important thing in knowing how many births there will be?

\# of Animals avaliable to breed — Fertility or Fecundity.

TO START — assume a constant birth rate.

$$\text{Births} = \text{\# of Births per Animal} \times \text{\# of Animals} = bP(n)$$

b = constant

similarly

$$\text{Deaths} = \text{rate at which death occur per animal} \times \text{\# of Animals} = dP(n)$$

d = constant

PLUG INTO SIMPLE MODEL

$$\begin{aligned} P(n+1) &= P(n) + bP(n) - dP(n) \\ &= P(n) + (b-d)P(n) \\ &= P(n) + rP(n) \end{aligned}$$

we call $r = b - d$ a growth/decay rate.

$r > 0$ more births than deaths

$r < 0$ more deaths than births.

NOTE Be careful of units... if n is measured in months then r is the growth rate over one month.

$$\boxed{\begin{aligned} P(n+1) &= P(n) + rP(n) \\ P(0) &= P_0 \end{aligned}}$$

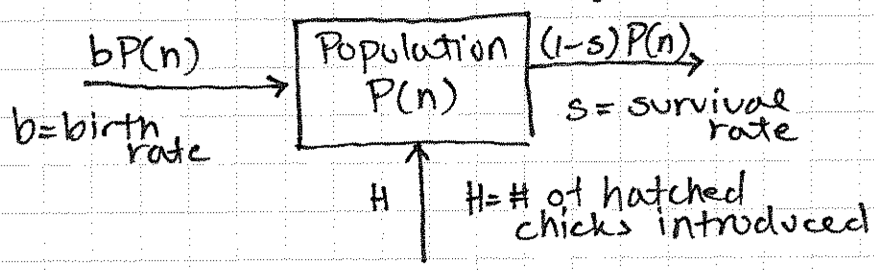
general constant growth Pop. Model.

Hand out and discuss Project 2 — Time at end of class to meet up w/ groups and Brainstorm.

FIRST ... more with COMPARTMENTAL DIAGRAMS

- These really are a wonderful tool for building complicated models!

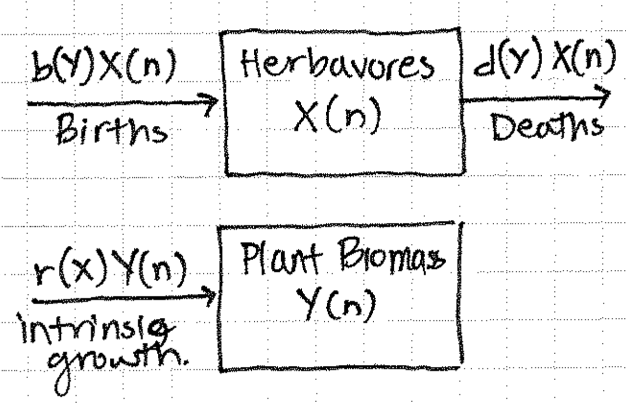
Ex Write a difference eqn and recurrence relation for the following system:



$P(n)$ = population of wild mountain chickens at year n.

$P(n+1) = P(n) + bP(n) - (1-s)P(n) + H$

Ex TWO SPECIES



What is hapening here?

NOTE $r(x) = b(x) - d(x)$ and it could be negative. Usually we put it as an inward arrow to signify $r > 0 \Rightarrow$ growth.

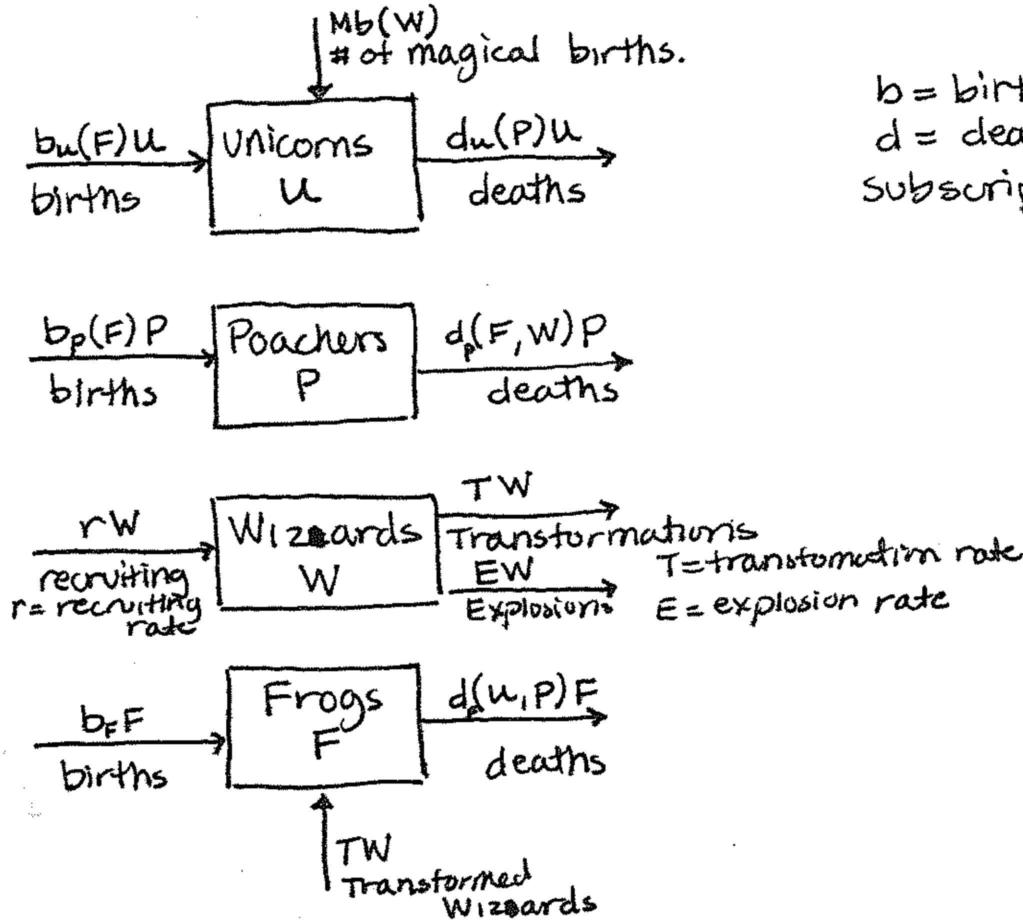
This gives us a system of eqns... we can still handle this on a spreadsheet!

$$X(n+1) = X(n) + b(Y)X(n) - d(Y)X(n)$$
$$Y(n+1) = Y(n) + r(x)Y(n)$$

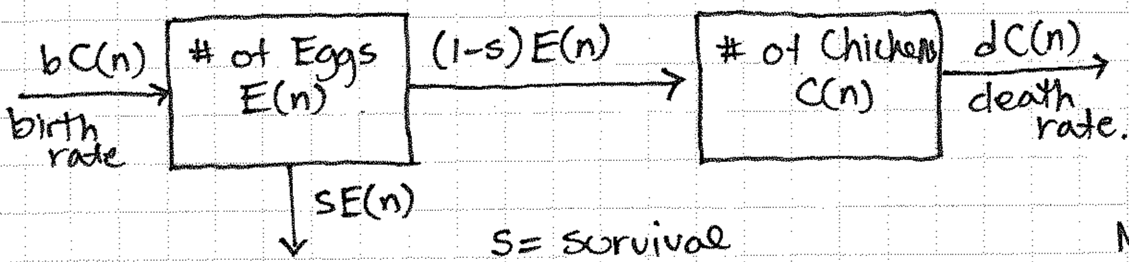
These must be solved step by step together!

Ex $\rightarrow$ Crazy story of Compartmental Diagrams.

b = birthrate
d = deathrate
subscript denotes species.



WE ALSO SOMETIMES DO THIS:



n = months

s = survival or hatching rate

MORE OF A FLOW DIAGRAM

$$E(n+1) - E(n) = bC(n) - sE(n) - (1-s)E(n)$$

$$C(n+1) - C(n) = (1-s)E(n) - dC(n)$$

SIMPLIFYING...

$$E(n+1) = E(n) + bC(n) - E(n)$$
$$= bC(n)$$

what does this mean?

$$C(n+1) = C(n) + (1-s)E(n) - dC(n).$$

YOUR DIAGRAMS AND EQUATIONS SHOULD MAKE SENSE !!

NEXT TIME ... some really powerful mathematical analysis.

TAKE THE REST OF CLASS TO MEET W/ YOUR GROUP AND START PROJECT 2.